Modello di linearizzazione della RG

Giampaolo Bottoni - 11 aprile 2010

Propongo un modo innovativo per linearizzare (in modo NON lineare) le equazioni della Relatività Generale.
Con questo modello potrei studiare le interazioni gravitazionali tra buchi neri ed il pregio del modello è che, nel caso in cui uno solo dei buchi ner sia nettamente prevalente su tutti gli altri, le equazioni ottenute tendono ad essere quelle esatte ossia tali per cui il tensore di Ricci risulti ovunque nullo.
Nelle vicinanze di un singolo buco nero l'azione del buco nero è nettamente prevalente rispetto a quelle di tutti gli altri e dunque questo modello fornisce una ottima approssimazione delle equazioni della RG tanto migliore quanto più la RG si discosta dal classico modello gravitazionale newtoniano.

```
(%i1) (if atom(lg) then load(ctensor));
(%o1)
C:/Programmi/Maxima-5.20.1/share/maxima/5.20.1/share/tensor/ctensor.mac
```

Inizializzo la libreria ctensor di Maxima.

```
(%i2) init_ctensor();
(%o2) done
```

Definisco una funzione del tutto generica delle coordinate oltre alla funzi di servizio rq.

```
(%i3) rq:x^2+y^2+z^2;
(%o3) z^2+y^2+x^2
```

La funzione U(x,y,z) del tutto generica è ricavata dalla classica funzione potenziale newtoniana. Specifico genericamente le variabili da cui dipende.

```
(%i4) depends(U,[x,y,z]);
(%o4) [U(x,y,z)]
```

La funzione b mi serve solo per semplificare l'input.

```
(%i5) b:U/(U-1);
(%o5) U/(U-1)
```

```
(%i6) lgmia: matrix (
      [ 1-U, 0,0,0 ],
      [ 0,-1+b*x^2/rq,b*x*y/rq, b*x*z/rq ],
      [ 0,b*x*y/rq,-1+b*y^2/rq,b*y*z/rq ],
      [ 0,b*x*z/rq,b*y*z/rq,-1+b*z^2/rq]);
```

(%o6)
$$\begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & \frac{x^2\,U}{(z^2+y^2+x^2)\,(U-1)}-1 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{x\,z\,U}{(z^2+y^2+x^2)\,(U-1)} \\ 0 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{y^2\,U}{(z^2+y^2+x^2)\,(U-1)}-1 & \frac{y\,z\,U}{(z^2+y^2+x^2)\,(U-1)} \\ 0 & \frac{x\,z\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{z^2\,U}{(z^2+y^2+x^2)\,(U-1)}-1 \end{bmatrix}$$

Dichiaro alla libreria ctensor che faccio uso di coordinate cartesiane

```
(%i7) ct_coords: [t,x,y,z];
```

(%o7) $[t,x,y,z]$

In base alle esigenze della libreria ctensor di cui ho fatto il load, inizializzo il tensore metrico covariante che si deve chiamare lg.

```
(%i8) lg:ratsimp(lgmia);
```

(%o8)
$$\begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & -\frac{(z^2+y^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\frac{(z^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \frac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\frac{(y^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \end{bmatrix}$$

Calcolo la metrica ovvero il tensore metrico controvariante che deve essere la matrice inversa del tensore metrico covariante.

```
(%i9) cmetric();
```

(%o9) *done*

```
(%i10) uug:ug$
```

Cerco di semplificare il piu' possibile l'espressione del tensore metrico controvariante

(%i11)
```
ug:ratsimp(uug);
```

(%o11)
$$\begin{bmatrix} -\frac{1}{U-1} & 0 & 0 & 0 \\ 0 & \frac{x^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \frac{x\,y\,U}{z^2+y^2+x^2} & \frac{x\,z\,U}{z^2+y^2+x^2} \\ 0 & \frac{x\,y\,U}{z^2+y^2+x^2} & \frac{y^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \frac{y\,z\,U}{z^2+y^2+x^2} \\ 0 & \frac{x\,z\,U}{z^2+y^2+x^2} & \frac{y\,z\,U}{z^2+y^2+x^2} & \frac{z^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} \end{bmatrix}$$

Anche se non serve ora faccio vedere che lg ed ug sono una la matrice inversa dell'altra

(%i12)
```
ratsimp(ug . lg);
```

(%o12)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ora calcolo i simboli di christoffel di prima e seconda specie visualizzandoli tutti. Attenzione alle regole della libreria ctensor che mette come terzo indice quello che di solito viene scritto come primo indice.

(%i13) christof(all);

(%t13) $$lcs_{1,1,2}=\frac{U_x}{2}$$

(%t14) $$lcs_{1,1,3}=\frac{U_y}{2}$$

(%t15) $$lcs_{1,1,4}=\frac{U_z}{2}$$

(%t16) $$lcs_{1,2,1}=-\frac{U_x}{2}$$

(%t17) $$lcs_{1,3,1}=-\frac{U_y}{2}$$

(%t18) $$lcs_{1,4,1}=-\frac{U_z}{2}$$

(%t19) $$lcs_{2,2,2}=\frac{((z^2+y^2)U-z^2-y^2-x^2)((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{(z^2+y^2)(U_x)-2\,x}{(z^2+y^2+x^2)U-z^2-y^2-x^2}}{2}$$

(%t20) $$lcs_{2,2,3}=(-\frac{((z^2+y^2)U-z^2-y^2-x^2)((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{(z^2+y^2)(U_y)+2\,y\,U-2\,y}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{2\,x\,y\,U((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{2\,x\,y(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{2\,y\,U}{(z^2+y^2+x^2)U-z^2-y^2-x^2})/2$$

(%t21) $$lcs_{2,2,4}=(-\frac{((z^2+y^2)U-z^2-y^2-x^2)((z^2+y^2+x^2)(U_z)+2\,z\,U-2\,z)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{(z^2+y^2)(U_z)+2\,z\,U-2\,z}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{2\,x\,z\,U((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{2\,x\,z(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{2\,z\,U}{(z^2+y^2+x^2)U-z^2-y^2-x^2})/2$$

(%t22) $$lcs_{2,3,2}=\frac{((z^2+y^2)U-z^2-y^2-x^2)((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{(z^2+y^2)(U_y)+2\,y\,U-2\,y}{(z^2+y^2+x^2)U-z^2-y^2-x^2}}{2}$$

(%t23) $$lcs_{2,3,3}=\frac{((z^2+x^2)U-z^2-y^2-x^2)((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{(z^2+x^2)(U_x)+2\,x\,U-2\,x}{(z^2+y^2+x^2)U-z^2-y^2-x^2}}{2}$$

(%t24) $$lcs_{2,3,4}=(\frac{x\,y\,U((z^2+y^2+x^2)(U_z)+2\,z\,U-2\,z)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{x\,y(U_z)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{x\,z\,U((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{x\,z(U_y)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-$$

Provo a semplificare una delle componenti del simbolo di Christoffel
di seconda specie.

```
(%i61) ratsimp(mcs[2,2,2]);
```

(%o61) $(x^3\,z\,U(U_z)+x^3\,y\,U(U_y)+((-2\,x^2\,z^2-2\,x^2\,y^2-x^4)\,U+x^2\,z^2+x^2\,y^2+x^4)(U_x)+(2\,x\,z^2+2\,x\,y^2)\,U^3+(-4\,x\,z^2-4\,x\,y^2)\,U^2+(2\,x\,z^2+2\,x\,y^2)\,U)/(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^2+(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4)\,U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)$

Creo il tensore di Riemann sopprimendo l'output che probabilmente sarebbe
troppo lungo per essere stampato.

```
(%i62) riemann(false);
```

(%o62) *done*

Ora faccio lo stesso calcolando il tensore di Ricci.

```
(%i63) ricci(false);
```

(%o63) *done*

Ora trasformo il tensore di Ricci in matrice.

```
(%i64) mat_ricci: ratsimp( matrix(
       [ ric[1,1],ric[1,2],ric[1,3],ric[1,4]],
       [ ric[2,1],ric[2,2],ric[2,3],ric[2,4]],
       [ ric[3,1],ric[3,2],ric[3,3],ric[3,4]],
       [ ric[4,1],ric[4,2],ric[4,3],ric[4,4]]))$
```

Innanzi tutto controllo che i termini non diagonali della
prima riga e colonna siano degli zeri.

```
(%i65) forsezeri: [ratsimp(ric[2,1]),ratsimp(ric[3,1]),ratsimp(ric[4,1]),
       ratsimp(ric[1,2]),
       ratsimp(ric[1,3]),
       ratsimp(ric[1,4])];
```

(%o65) *[0,0,0,0,0,0]*

Controllo che il tensore di ricci e' un tensore simmetrico... come da manua
ma se uno vuol fare il san Tommaso...

```
(%i66) certozeri: [ratsimp(ric[2,3]-ric[3,2]), ratsimp(ric[2,4]-ric[4,2]),
       ratsimp(ric[3,4]-ric[4,3]) ];
```

(%o66) *[0,0,0]*

Questi non sono zeri ma se la funzione U(x,y,z) assume
dei valori adeguati lo debbono diventare...
Intanto li stampo...

(%i67) `ratsimp(ric[1,1]);`

(%o67)
$$((z^2U^2+(-2z^2-y^2-x^2)U+z^2+y^2+x^2)(U_{zz})+(y^2+x^2)(U_z)^2+(-2yz(U_y)-2xz(U_x)+2zU^2-2zU)(U_z)+(y^2U^2+(-z^2-2y^2-x^2)U+z^2+y^2+x^2)(U_{yy})+(2yzU^2-2yzU)(U_{yz})+(z^2+x^2)(U_y)^2+(-2xy(U_x)+2yU^2-2yU)(U_y)+(x^2U^2+(-z^2-y^2-2x^2)U+z^2+y^2+x^2)(U_{xx})+(2xzU^2-2xzU)(U_{xz})+(2xyU^2-2xyU)(U_{xy})+(z^2+y^2)(U_x)^2+(2xU^2-2xU)(U_x))/((2z^2+2y^2+2x^2)U-2z^2-2y^2-2x^2)$$

(%i68) `ratsimp(ric[2,2]);`

(%o68)
$$((x^2z^2U^2+(-2x^2z^2-x^2y^2-x^4)U+x^2z^2+x^2y^2+x^4)(U_{zz})+(-x^2z^2U+x^2z^2+x^2y^2+x^4)(U_z)^2+(-2x^2yzU(U_y)+((2xz^3+2xy^2z)U-2xz^3+(-2xy^2-2x^3)z)(U_x)+(2z^3+2y^2z)U^3+(-6z^3-6y^2z)U^2+(6z^3+(6y^2+2x^2)z)U-2z^3+(-2y^2-2x^2)z)(U_z)+(x^2y^2U^2+(-x^2z^2-2x^2y^2-x^4)U+x^2z^2+x^2y^2+x^4)(U_{yy})+(2x^2yzU^2-2x^2yzU)(U_{yz})+(-x^2y^2U+x^2z^2+x^2y^2+x^4)(U_y)^2+((2xyz^2+2xy^3)U-2xyz^2-2xy^3-2x^3y)(U_x)+(2yz^2+2y^3)U^3+(-6yz^2-6y^3)U^2+(6yz^2+6y^3+2x^2y)U-2yz^2-2y^3-2x^2y)(U_y)+((-2x^2z^2-2x^2y^2-x^4)U^2+(3x^2z^2+3x^2y^2+2x^4)U-x^2z^2-x^2y^2-x^4)(U_{xx})+((-2xz^3-2xy^2z)U^2+(4xz^3+(4xy^2+2x^3)z)U-2xz^3+(-2xy^2-2x^3)z)(U_{xz})+((-2xyz^2-2xy^3)U^2+(4xyz^2+4xy^3+2x^3y)U-2xyz^2-2xy^3-2x^3y)(U_{xy})+((-z^4-2y^2z^2-y^4)U+z^4+(2y^2+x^2)z^2+y^4+x^2y^2)(U_x)^2+((2xz^2+2xy^2)U^3+(-8xz^2-8xy^2-2x^3)U^2+(10xz^2+10xy^2+6x^3)U-4xz^2-4xy^2-4x^3)(U_x)+(2z^2+2y^2)U^4+(-6z^2-6y^2)U^3+(6z^2+6y^2)U^2+(-2z^2-2y^2)U)/((2z^4+(4y^2+4x^2)z^2+2y^4+4x^2y^2+2x^4)U^3+(-6z^4+(-12y^2-12x^2)z^2-6y^4-12x^2y^2-6x^4)U^2+(6z^4+(12y^2+12x^2)z^2+6y^4+12x^2y^2+6x^4)U-2z^4+(-4y^2-4x^2)z^2-2y^4-4x^2y^2-2x^4)$$

```
(%i69) ratsimp(ric[3,3]);
```

(%o69)

$$((y^2 z^2 U^2+(-2 y^2 z^2-y^4-x^2 y^2) U+y^2 z^2+y^4+x^2 y^2)(U_{zz})+$$
$$(-y^2 z^2 U+y^2 z^2+y^4+x^2 y^2)(U_z)^2+(((2 y z^3+2 x^2 y z) U-2 y z^3+(-2 y^3-2 x^2 y) z)$$
$$(U_y)-2 x y^2 z U(U_x)+(2 z^3+2 x^2 z) U^3+(-6 z^3-6 x^2 z) U^2+(6 z^3+(2 y^2+6 x^2) z) U-2$$
$$z^3+(-2 y^2-2 x^2) z)(U_z)+$$
$$((-2 y^2 z^2-y^4-2 x^2 y^2) U^2+(3 y^2 z^2+2 y^4+3 x^2 y^2) U-y^2 z^2-y^4-x^2 y^2)(U_{yy})+$$
$$((-2 y z^3-2 x^2 y z) U^2+(4 y z^3+(2 y^3+4 x^2 y) z) U-2 y z^3+(-2 y^3-2 x^2 y) z)(U_{yz})+$$
$$((-z^4-2 x^2 z^2-x^4) U+z^4+(y^2+2 x^2) z^2+x^2 y^2+x^4)(U_y)^2+($$
$$((2 x y z^2+2 x^3 y) U-2 x y z^2-2 x y^3-2 x^3 y)(U_x)+(2 y z^2+2 x^2 y) U^3+$$
$$(-8 y z^2-2 y^3-8 x^2 y) U^2+(10 y z^2+6 y^3+10 x^2 y) U-4 y z^2-4 y^3-4 x^2 y)(U_y)+$$
$$(x^2 y^2 U^2+(-y^2 z^2-y^4-2 x^2 y^2) U+y^2 z^2+y^4+x^2 y^2)(U_{xx})+(2 x y^2 z U^2-2 x y^2 z U)$$
$$(U_{xz})+((-2 x y z^2-2 x^3 y) U^2+(4 x y z^2+2 x y^3+4 x^3 y) U-2 x y z^2-2 x y^3-2 x^3 y)$$
$$(U_{xy})+(-x^2 y^2 U+y^2 z^2+y^4+x^2 y^2)(U_x)^2+$$
$$((2 x z^2+2 x^3) U^3+(-6 x z^2-6 x^3) U^2+(6 x z^2+2 x y^2+6 x^3) U-2 x z^2-2 x y^2-2 x^3)$$
$$(U_x)+(2 z^2+2 x^2) U^4+(-6 z^2-6 x^2) U^3+(6 z^2+6 x^2) U^2+(-2 z^2-2 x^2) U)/($$
$$(2 z^4+(4 y^2+4 x^2) z^2+2 y^4+4 x^2 y^2+2 x^4) U^3+$$
$$(-6 z^4+(-12 y^2-12 x^2) z^2-6 y^4-12 x^2 y^2-6 x^4) U^2+$$
$$(6 z^4+(12 y^2+12 x^2) z^2+6 y^4+12 x^2 y^2+6 x^4) U-2 z^4+(-4 y^2-4 x^2) z^2-2 y^4-4 x^2$$
$$y^2-2 x^4)$$

```
(%i70) ratsimp(ric[4,4]);
```

$$
\begin{aligned}
(\%o70)\ & -(((z^4+(2y^2+2x^2)z^2)U^2+((-3y^2-3x^2)z^2-2z^4)U+z^4+(y^2+x^2)z^2)\\
&(U_{zz})+((y^4+2x^2y^2+x^4)U+(-y^2-x^2)z^2-y^4-2x^2y^2-x^4)(U_z)^2+(\\
&((-2y^3-2x^2y)zU+2yz^3+(2y^3+2x^2y)z)(U_y)+\\
&((-2xy^2-2x^3)zU+2xz^3+(2xy^2+2x^3)z)(U_x)+(-2y^2-2x^2)zU^3+\\
&(2z^3+(8y^2+8x^2)z)U^2+((-10y^2-10x^2)z-6z^3)U+4z^3+(4y^2+4x^2)z)(U_z)+\\
&(-y^2z^2U^2+(z^4+(2y^2+x^2)z^2)U-z^4+(-y^2-x^2)z^2)(U_{yy})+\\
&((2y^3+2x^2y)zU^2+((-4y^3-4x^2y)z-2yz^3)U+2yz^3+(2y^3+2x^2y)z)(U_{yz})+\\
&(y^2z^2U-z^4+(-y^2-x^2)z^2)(U_y)^2+(2xyz^2U(U_x)+(-2y^3-2x^2y)U^3+\\
&(6y^3+6x^2y)U^2+(-2yz^2-6y^3-6x^2y)U+2yz^2+2y^3+2x^2y)(U_y)+\\
&(-x^2z^2U^2+(z^4+(y^2+2x^2)z^2)U-z^4+(-y^2-x^2)z^2)(U_{xx})+\\
&((2xy^2+2x^3)zU^2+((-4xy^2-4x^3)z-2xz^3)U+2xz^3+(2xy^2+2x^3)z)(U_{xz})+\\
&(2xyz^2U-2xyz^2U^2)(U_{xy})+(x^2z^2U-z^4+(-y^2-x^2)z^2)(U_x)^2+\\
&((-2xy^2-2x^3)U^3+(6xy^2+6x^3)U^2+(-2xz^2-6xy^2-6x^3)U+2xz^2+2xy^2+2x^3)\\
&(U_x)+(-2y^2-2x^2)U^4+(6y^2+6x^2)U^3+(-6y^2-6x^2)U^2+(2y^2+2x^2)U)/(\\
&(2z^4+(4y^2+4x^2)z^2+2y^4+4x^2y^2+2x^4)U^3+\\
&(-6z^4+(-12y^2-12x^2)z^2-6y^4-12x^2y^2-6x^4)U^2+\\
&(6z^4+(12y^2+12x^2)z^2+6y^4+12x^2y^2+6x^4)U-2z^4+(-4y^2-4x^2)z^2-2y^4-4x^2\\
&y^2-2x^4)
\end{aligned}
$$

```
(%i71) ratsimp(ric[2,3]);
```

(%o71)

$$((x\,y\,z^2\,U^2+(-2\,x\,y\,z^2-x\,y^3-x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_{z\,z})+$$
$$(-x\,y\,z^2\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_z)^2+(((x\,z^3+(x^3-x\,y^2)\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)$$
$$(U_y)+((y\,z^3+(y^3-x^2\,y)\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_x)-2\,x\,y\,z\,U^3+6\,x\,y\,z\,U^2-4\,x\,y$$
$$z\,U)\,(U_z)+((-x\,y\,z^2-x^3\,y)\,U^2+(x\,y\,z^2+x^3\,y)\,U)(U_{y\,y})+$$
$$(((x\,y^2-x^3)\,z-x\,z^3)\,U^2+(2\,x\,z^3+2\,x^3\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)(U_{y\,z})+(x\,y\,z^2+x^3\,y)$$
$$U\,(U_y)^2+(((-z^4+(-y^2-x^2)\,z^2-2\,x^2\,y^2)\,U+z^4+(y^2+x^2)\,z^2)(U_x)-2\,x\,y^2\,U^3+$$
$$(-x\,z^2+5\,x\,y^2-x^3)\,U^2+(2\,x\,z^2-2\,x\,y^2+2\,x^3)\,U-x\,z^2-x\,y^2-x^3)\,(U_y)+$$
$$((-x\,y\,z^2-x\,y^3)\,U^2+(x\,y\,z^2+x\,y^3)\,U)(U_{x\,x})+$$
$$(((x^2\,y-y^3)\,z-y\,z^3)\,U^2+(2\,y\,z^3+2\,y^3\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_{x\,z})+($$
$$((-y^2-x^2)\,z^2-y^4-x^4)\,U^2+((2\,y^2+2\,x^2)\,z^2+2\,y^4+2\,x^2\,y^2+2\,x^4)\,U+(-y^2-x^2)\,z^2-y^4$$
$$-2\,x^2\,y^2-x^4)\,(U_{x\,y})+(x\,y\,z^2+x\,y^3)\,U\,(U_x)^2+$$
$$(-2\,x^2\,y\,U^3+(-y\,z^2-y^3+5\,x^2\,y)\,U^2+(2\,y\,z^2+2\,y^3-2\,x^2\,y)\,U-y\,z^2-y^3-x^2\,y)(U_x)-2$$
$$x\,y\,U^4+6\,x\,y\,U^3-6\,x\,y\,U^2+2\,x\,y\,U)\,/\,((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^3$$
$$+(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4)\,U^2+$$
$$(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4)\,U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2$$
$$y^2-2\,x^4)$$

```
(%i72) ratsimp(ric[2,4]);
```

(%o72)

$$-(((x y^2+x^3) z U^2+(-x y^2-x^3) z U)(U_{zz})+(-x y^2-x^3) z U (U_z)^2+(((x y z^2-x y^3-x^3 y) U+x y z^2+x y^3+x^3 y)(U_y)+(((y^2+2 x^2) z^2+y^4+x^2 y^2) U-y^2 z^2-y^4-x^2 y^2)(U_x)+2 x z^2 U^3+(-5 x z^2+x y^2+x^3) U^2+(2 x z^2-2 x y^2-2 x^3) U+x z^2+x y^2+x^3)(U_z)+(-x y^2 z U^2+(x z^3+(2 x y^2+x^3) z) U-x z^3+(-x y^2-x^3) z)(U_{yy})+((-x y z^2+x y^3+x^3 y) U^2+(-2 x y^3-2 x^3 y) U+x y z^2+x y^3+x^3 y)(U_{yz})+(x y^2 z U-x z^3+(-x y^2-x^3) z)(U_y)^2+\left(((x^2 y-y^3) z-y z^3) U+y z^3+(y^3+x^2 y) z)(U_x)+2 x y z U^3-6 x y z U^2+4 x y z U\right)(U_y)+((x z^3+x y^2 z) U^2+(-x z^3-x y^2 z) U)(U_{xx})+((z^4+y^2 z^2+x^2 y^2+x^4) U^2+(-2 z^4+(-2 y^2-2 x^2) z^2-2 x^2 y^2-2 x^4) U+z^4+(y^2+2 x^2) z^2+x^2 y^2+x^4)(U_{xz})+((y z^3+(y^3-x^2 y) z) U^2+(-2 y z^3-2 y^3 z) U+y z^3+(y^3+x^2 y) z)(U_{xy})+(-x z^3-x y^2 z) U (U_x)^2+(2 x^2 z U^3+(z^3+(y^2-5 x^2) z) U^2+((2 x^2-2 y^2) z-2 z^3) U+z^3+(y^2+x^2) z)(U_x)+2 x z U^4-6 x z U^3+6 x z U^2-2 x z U)/((2 z^4+(4 y^2+4 x^2) z^2+2 y^4+4 x^2 y^2+2 x^4) U^3+(-6 z^4+(-12 y^2-12 x^2) z^2-6 y^4-12 x^2 y^2-6 x^4) U^2+(6 z^4+(12 y^2+12 x^2) z^2+6 y^4+12 x^2 y^2+6 x^4) U-2 z^4+(-4 y^2-4 x^2) z^2-2 y^4-4 x^2 y^2-2 x^4)$$

(%i73) ratsimp(ric[3,4]);

(%o73) $$-(((y^3+x^2\,y)\,z\,U^2+(-y^3-x^2\,y)\,z\,U)(U_{zz})+(-y^3-x^2\,y)\,z\,U\,(U_z)^2+(((2\,y^2+x^2)\,z^2+x^2\,y^2+x^4)\,U-x^2\,z^2-x^2\,y^2-x^4)(U_y)+((x\,y\,z^2-x\,y^3-x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_x)+2\,y\,z^2\,U^3+(-5\,y\,z^2+y^3+x^2\,y)\,U^2+(2\,y\,z^2-2\,y^3-2\,x^2\,y)\,U+y\,z^2+y^3+x^2\,y)\,(U_z)+((y\,z^3+x^2\,y\,z)\,U^2+(-y\,z^3-x^2\,y\,z)\,U)(U_{yy})+((z^4+x^2\,z^2+y^4+x^2\,y^2)\,U^2+(-2\,z^4+(-2\,y^2-2\,x^2)\,z^2-2\,y^4-2\,x^2\,y^2)\,U+z^4+(2\,y^2+x^2)\,z^2+y^4+x^2\,y^2)\,(U_{yz})+(-y\,z^3-x^2\,y\,z)\,U\,(U_y)^2+(((x\,y^2-x^3)\,z-x\,z^3)\,U+x\,z^3+(x\,y^2+x^3)\,z)(U_x)+2\,y^2\,z\,U^3+(z^3+(x^2-5\,y^2)\,z)\,U^2+((2\,y^2-2\,x^2)\,z-2\,z^3)\,U+z^3+(y^2+x^2)\,z)\,(U_y)+(-x^2\,y\,z\,U^2+(y\,z^3+(y^3+2\,x^2\,y)\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_{xx})+((-x\,y\,z^2+x\,y^3+x^3\,y)\,U^2+(-2\,x\,y^3-2\,x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_{xz})+((x\,z^3+(x^3-x\,y^2)\,z)\,U^2+(-2\,x\,z^3-2\,x^3\,z)\,U+x\,z^3+(x\,y^2+x^3)\,z)(U_{xy})+(x^2\,y\,z\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_x)^2+(2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U)(U_x)+2\,y\,z\,U^4-6\,y\,z\,U^3+6\,y\,z\,U^2-2\,y\,z\,U)\,/\,((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^3+(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4)\,U^2+(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4)\,U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

Ora mi preparo a specificare cosa deve valere la funzione U(x,y,z) per fare in modo che la metrica sia quella di un buco nero neutro ossia sia la metri di Schwarzschild.

(%i74) v0n:2*m/rq^(1/2);

(%o74) $$\frac{2\,m}{\sqrt{z^2+y^2+x^2}}$$

Specifico tutte le derivate di primo e secondo ordine.

(%i75) vxn:ratsimp(diff(v0n,x));

(%o75) $$-\frac{2\,m\,x}{(z^2+y^2+x^2)^{3/2}}$$

(%i76) vyn:ratsimp(diff(v0n,y))$

(%i77) vzn:ratsimp(diff(v0n,z))$

(%i78) vxxn:ratsimp(diff(vxn,x));

(%o78) $$-\frac{\sqrt{z^2+y^2+x^2}\,(2\,m\,z^2+2\,m\,y^2-4\,m\,x^2)}{z^6+(3\,y^2+3\,x^2)\,z^4+(3\,y^4+6\,x^2\,y^2+3\,x^4)\,z^2+y^6+3\,x^2\,y^4+3\,x^4\,y^2+x^6}$$

(%i79) vyyn:ratsimp(diff(vyn,y))$

```
(%i80) vzzn:ratsimp(diff(vzn,z))$
```

```
(%i81) vxyn:ratsimp(diff(vxn,y));
```

$$(\%o81)\quad \frac{6\,m\,x\,y}{\sqrt{z^2+y^2+x^2}\,(z^4+(2\,y^2+2\,x^2)\,z^2+y^4+2\,x^2\,y^2+x^4)}$$

```
(%i82) vxzn:ratsimp(diff(vxn,z))$
```

```
(%i83) vyzn:ratsimp(diff(vyn,z))$
```

Ora sostituisco alle derivate alcuni simboli ossia uso v0, vx, vy, vz, vxx, vxy, vzz, vxy, vxz, vyz.
Inizio la procedura con ric[1,1]

```
(%i84) rics:ratsimp(ric[1,1]);
```

$$(\%o84)\quad ((z^2\,U^2+(-2\,z^2-y^2-x^2)\,U+z^2+y^2+x^2)(U_{z\,z})+(y^2+x^2)(U_z)^2+\left(-2\,y\,z\,(U_y)-2\,x\,z\,(U_x)+2\,z\,U^2-2\,z\,U\right)(U_z)+(y^2\,U^2+(-z^2-2\,y^2-x^2)\,U+z^2+y^2+x^2)(U_{y\,y})+(2\,y\,z\,U^2-2\,y\,z\,U)(U_{y\,z})+(z^2+x^2)(U_y)^2+\left(-2\,x\,y\,(U_x)+2\,y\,U^2-2\,y\,U\right)(U_y)+(x^2\,U^2+(-z^2-y^2-2\,x^2)\,U+z^2+y^2+x^2)(U_{x\,x})+(2\,x\,z\,U^2-2\,x\,z\,U)(U_{x\,z})+(2\,x\,y\,U^2-2\,x\,y\,U)(U_{x\,y})+(z^2+y^2)(U_x)^2+(2\,x\,U^2-2\,x\,U)(U_x))\,/\,((2\,z^2+2\,y^2+2\,x^2)\,U-2\,z^2-2\,y^2-2\,x^2)$$

```
(%i85) ric1:ratsimp(subst(vx,diff(U,x),rics))$
```

```
(%i86) ric2:ratsimp(subst(vy,diff(U,y),ric1))$
```

```
(%i87) ric3:ratsimp(subst(vz,diff(U,z),ric2))$
```

```
(%i88) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$
```

```
(%i89) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$
```

```
(%i90) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$
```

```
(%i91) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$
```

```
(%i92) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$
```

```
(%i93) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$
```

```
(%i94) rica:ratsimp(subst(v0,U,ric9));
```

$$
\begin{aligned}
(\%o94)\ & (((v0^2-2\,v0+1)\,vzz+(1-v0)\,vyy+vy^2+(1-v0)\,vxx+vx^2)\,z^2+\\
& (((2\,v0^2-2\,v0)\,vyz-2\,vy\,vz)\,y+((2\,v0^2-2\,v0)\,vxz-2\,vx\,vz)\,x+(2\,v0^2-2\,v0)\,vz)\,z+\\
& ((1-v0)\,vzz+vz^2+(v0^2-2\,v0+1)\,vyy+(1-v0)\,vxx+vx^2)\,y^2+\\
& (((2\,v0^2-2\,v0)\,vxy-2\,vx\,vy)\,x+(2\,v0^2-2\,v0)\,vy)\,y+\\
& ((1-v0)\,vzz+vz^2+(1-v0)\,vyy+vy^2+(v0^2-2\,v0+1)\,vxx)\,x^2+(2\,v0^2-2\,v0)\,vx\,x)/(\\
& (2\,v0-2)\,z^2+(2\,v0-2)\,y^2+(2\,v0-2)\,x^2)
\end{aligned}
$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i95) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i96) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i97) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i98) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i99) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i100) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i101) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i102) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i103) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i104) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i105) ratsimp(rican);
(%o105) 0
```

Ripeto tutta la procedura con ric[2,2]

(%i106) `rics:ratsimp(ric[2,2]);`

(%o106)
$$((x^2 z^2 U^2+(-2 x^2 z^2-x^2 y^2-x^4) U+x^2 z^2+x^2 y^2+x^4)(U_{z z})+$$
$$(-x^2 z^2 U+x^2 z^2+x^2 y^2+x^4)(U_z)^2+(-2 x^2 y z U(U_y)+$$
$$((2 x z^3+2 x y^2 z) U-2 x z^3+(-2 x y^2-2 x^3) z)(U_x)+(2 z^3+2 y^2 z) U^3+$$
$$(-6 z^3-6 y^2 z) U^2+(6 z^3+(6 y^2+2 x^2) z) U-2 z^3+(-2 y^2-2 x^2) z)(U_z)+$$
$$(x^2 y^2 U^2+(-x^2 z^2-2 x^2 y^2-x^4) U+x^2 z^2+x^2 y^2+x^4)(U_{y y})+(2 x^2 y z U^2-2 x^2 y z U)$$
$$(U_{y z})+(-x^2 y^2 U+x^2 z^2+x^2 y^2+x^4)(U_y)^2+($$
$$((2 x y z^2+2 x y^3) U-2 x y z^2-2 x y^3-2 x^3 y)(U_x)+(2 y z^2+2 y^3) U^3+$$
$$(-6 y z^2-6 y^3) U^2+(6 y z^2+6 y^3+2 x^2 y) U-2 y z^2-2 y^3-2 x^2 y)(U_y)+$$
$$((-2 x^2 z^2-2 x^2 y^2-x^4) U^2+(3 x^2 z^2+3 x^2 y^2+2 x^4) U-x^2 z^2-x^2 y^2-x^4)(U_{x x})+$$
$$((-2 x z^3-2 x y^2 z) U^2+(4 x z^3+(4 x y^2+2 x^3) z) U-2 x z^3+(-2 x y^2-2 x^3) z)(U_{x z})+$$
$$((-2 x y z^2-2 x y^3) U^2+(4 x y z^2+4 x y^3+2 x^3 y) U-2 x y z^2-2 x y^3-2 x^3 y)(U_{x y})+$$
$$((-z^4-2 y^2 z^2-y^4) U+z^4+(2 y^2+x^2) z^2+y^4+x^2 y^2)(U_x)^2+((2 x z^2+2 x y^2) U^3+$$
$$(-8 x z^2-8 x y^2-2 x^3) U^2+(10 x z^2+10 x y^2+6 x^3) U-4 x z^2-4 x y^2-4 x^3)(U_x)+$$
$$(2 z^2+2 y^2) U^4+(-6 z^2-6 y^2) U^3+(6 z^2+6 y^2) U^2+(-2 z^2-2 y^2) U)/($$
$$(2 z^4+(4 y^2+4 x^2) z^2+2 y^4+4 x^2 y^2+2 x^4) U^3+$$
$$(-6 z^4+(-12 y^2-12 x^2) z^2-6 y^4-12 x^2 y^2-6 x^4) U^2+$$
$$(6 z^4+(12 y^2+12 x^2) z^2+6 y^4+12 x^2 y^2+6 x^4) U-2 z^4+(-4 y^2-4 x^2) z^2-2 y^4-4 x^2$$
$$y^2-2 x^4)$$

(%i107) `ric1:ratsimp(subst(vx,diff(U,x),rics))$`

(%i108) `ric2:ratsimp(subst(vy,diff(U,y),ric1))$`

(%i109) `ric3:ratsimp(subst(vz,diff(U,z),ric2))$`

(%i110) `ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$`

(%i111) `ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$`

(%i112) `ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$`

(%i113) `ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$`

(%i114) `ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$`

(%i115) `ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$`

(%i116) rica:ratsimp(subst(v0,U,ric9));

(%o116)
$$-((v0-1)\,vx^2\,z^4+(((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz)\,z^3+((2\,v0-2)\,vx^2\,y^2+(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy)\,y+((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(v0-1)\,vyy-vy^2+(2\,v0^2-3\,v0+1)\,vxx-vx^2)\,x^2+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,z^2+(((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz)\,y^2+(2\,v0\,vy\,vz+(2\,v0-2\,v0^2)\,vyz)\,x^2\,y+(2\,vx\,vz+(2-2\,v0)\,vxz)\,x^3+(2-2\,v0)\,vz\,x^2)\,z+(v0-1)\,vx^2\,y^4+(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy)\,y^3+((v0-1)\,vzz-vz^2+(-v0^2+2\,v0-1)\,vyy+(v0-1)\,vy^2+(2\,v0^2-3\,v0+1)\,vxx-vx^2)\,x^2+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,y^2+((2\,vx\,vy+(2-2\,v0)\,vxy)\,x^3+(2-2\,v0)\,vy\,x^2)\,y+((v0-1)\,vzz-vz^2+(v0-1)\,vyy-vy^2+(v0^2-2\,v0+1)\,vxx)\,x^4+(2\,v0^2-6\,v0+4)\,vx\,x^3)/((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i117) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i118) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i119) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i120) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i121) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i122) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i123) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i124) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i125) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i126) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i127) ratsimp(rican);
(%o127) 0
```

Ripeto tutta la procedura con ric[3,3]

```
(%i128) rics:ratsimp(ric[3,3]);
```

(%o128)

$$((y^2 z^2 U^2+(-2 y^2 z^2-y^4-x^2 y^2) U+y^2 z^2+y^4+x^2 y^2)(U_{zz})+(-y^2 z^2 U+y^2 z^2+y^4+x^2 y^2)(U_z)^2+(((2 y z^3+2 x^2 y z) U-2 y z^3+(-2 y^3-2 x^2 y) z)(U_y)-2 x y^2 z U(U_x)+(2 z^3+2 x^2 z) U^3+(-6 z^3-6 x^2 z) U^2+(6 z^3+(2 y^2+6 x^2) z) U-2 z^3+(-2 y^2-2 x^2) z)(U_z)+((-2 y^2 z^2-y^4-2 x^2 y^2) U^2+(3 y^2 z^2+2 y^4+3 x^2 y^2) U-y^2 z^2-y^4-x^2 y^2)(U_{yy})+((-2 y z^3-2 x^2 y z) U^2+(4 y z^3+(2 y^3+4 x^2 y) z) U-2 y z^3+(-2 y^3-2 x^2 y) z)(U_{yz})+((-z^4-2 x^2 z^2-x^4) U+z^4+(y^2+2 x^2) z^2+x^2 y^2+x^4)(U_y)^2+((2 x y z^2+2 x^3 y) U-2 x y z^2-2 x y^3-2 x^3 y)(U_x)+(2 y z^2+2 x^2 y) U^3+(-8 y z^2-2 y^3-8 x^2 y) U^2+(10 y z^2+6 y^3+10 x^2 y) U-4 y z^2-4 y^3-4 x^2 y)(U_y)+(x^2 y^2 U^2+(-y^2 z^2-y^4-2 x^2 y^2) U+y^2 z^2+y^4+x^2 y^2)(U_{xx})+(2 x y^2 z U^2-2 x y^2 z U)(U_{xz})+((-2 x y z^2-2 x^3 y) U^2+(4 x y z^2+2 x y^3+4 x^3 y) U-2 x y z^2-2 x y^3-2 x^3 y)(U_{xy})+(-x^2 y^2 U+y^2 z^2+y^4+x^2 y^2)(U_x)^2+((2 x z^2+2 x^3) U^3+(-6 x z^2-6 x^3) U^2+(6 x z^2+2 x y^2+6 x^3) U-2 x z^2-2 x y^2-2 x^3)(U_x)+(2 z^2+2 x^2) U^4+(-6 z^2-6 x^2) U^3+(6 z^2+6 x^2) U^2+(-2 z^2-2 x^2) U)/((2 z^4+(4 y^2+4 x^2) z^2+2 y^4+4 x^2 y^2+2 x^4) U^3+(-6 z^4+(-12 y^2-12 x^2) z^2-6 y^4-12 x^2 y^2-6 x^4) U^2+(6 z^4+(12 y^2+12 x^2) z^2+6 y^4+12 x^2 y^2+6 x^4) U-2 z^4+(-4 y^2-4 x^2) z^2-2 y^4-4 x^2 y^2-2 x^4)$$

```
(%i129) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i130) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i131) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i132) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i133) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i134) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i135) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i136) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$
```

```
(%i137) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$
```

```
(%i138) rica:ratsimp(subst(v0,U,ric9));
```

(%o138)

$$-((v0-1)\,vy^2\,z^4+(((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,y+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz)\,z^3+((((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(2\,v0^2-3\,v0+1)\,vyy-vy^2+(v0-1)\,vxx-vx^2)\,y^2+(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vy)\,y+(2\,v0-2)\,vy^2\,x^2+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,z^2+((2\,vy\,vz+(2-2\,v0)\,vyz)\,y^3+((2\,v0\,vx\,vz+(2\,v0-2\,v0^2)\,vxz)\,x+(2-2\,v0)\,vz)\,y^2+((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,x^2\,y+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz\,x^2)\,z+((v0-1)\,vzz-vz^2+(v0^2-2\,v0+1)\,vyy+(v0-1)\,vxx-vx^2)\,y^4+((2\,vx\,vy+(2-2\,v0)\,vxy)\,x+(2\,v0^2-6\,v0+4)\,vy)\,y^3+((v0-1)\,vzz-vz^2+(2\,v0^2-3\,v0+1)\,vyy-vy^2+(-v0^2+2\,v0-1)\,vxx+(v0-1)\,vx^2)\,x^2+(2-2\,v0)\,vx\,x)\,y^2+(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x^3+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vy\,x^2)\,y+(v0-1)\,vy^2\,x^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x^3+(-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,x^2)/((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i139) ric1n:ratsimp(subst(vxn,vx,rica))$
```

```
(%i140) ric2n:ratsimp(subst(vyn,vy,ric1n))$
```

```
(%i141) ric3n:ratsimp(subst(vzn,vz,ric2n))$
```

```
(%i142) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$
```

```
(%i143) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$
```

```
(%i144) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$
```

```
(%i145) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$
```

```
(%i146) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$
```

```
(%i147) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$
```

```
(%i148) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i149) ratsimp(rican);
```

(%o149) 0

Ripeto tutta la procedura con ric[4,4]

```
(%i150) rics:ratsimp(ric[4,4]);
```

(%o150)
$$-(((z^4+(2y^2+2x^2)z^2)U^2+((-3y^2-3x^2)z^2-2z^4)U+z^4+(y^2+x^2)z^2)(U_{zz})+((y^4+2x^2y^2+x^4)U+(-y^2-x^2)z^2-y^4-2x^2y^2-x^4)(U_z)^2+((-2y^3-2x^2y)zU+2yz^3+(2y^3+2x^2y)z)(U_y)+((-2xy^2-2x^3)zU+2xz^3+(2xy^2+2x^3)z)(U_x)+(-2y^2-2x^2)zU^3+(2z^3+(8y^2+8x^2)z)U^2+((-10y^2-10x^2)z-6z^3)U+4z^3+(4y^2+4x^2)z)(U_z)+(-y^2z^2U^2+(z^4+(2y^2+x^2)z^2)U-z^4+(-y^2-x^2)z^2)(U_{yy})+((2y^3+2x^2y)zU^2+((-4y^3-4x^2y)z-2yz^3)U+2yz^3+(2y^3+2x^2y)z)(U_{yz})+(y^2z^2U-z^4+(-y^2-x^2)z^2)(U_y)^2+(2xyz^2U(U_x)+(-2y^3-2x^2y)U^3+(6y^3+6x^2y)U^2+(-2yz^2-6y^3-6x^2y)U+2yz^2+2y^3+2x^2y)(U_y)+(-x^2z^2U^2+(z^4+(y^2+2x^2)z^2)U-z^4+(-y^2-x^2)z^2)(U_{xx})+((2xy^2+2x^3)zU^2+((-4xy^2-4x^3)z-2xz^3)U+2xz^3+(2xy^2+2x^3)z)(U_{xz})+(2xyz^2U-2xyz^2U^2)(U_{xy})+(x^2z^2U-z^4+(-y^2-x^2)z^2)(U_x)^2+((-2xy^2-2x^3)U^3+(6xy^2+6x^3)U^2+(-2xz^2-6xy^2-6x^3)U+2xz^2+2xy^2+2x^3)(U_x)+(-2y^2-2x^2)U^4+(6y^2+6x^2)U^3+(-6y^2-6x^2)U^2+(2y^2+2x^2)U)/((2z^4+(4y^2+4x^2)z^2+2y^4+4x^2y^2+2x^4)U^3+(-6z^4+(-12y^2-12x^2)z^2-6y^4-12x^2y^2-6x^4)U^2+(6z^4+(12y^2+12x^2)z^2+6y^4+12x^2y^2+6x^4)U-2z^4+(-4y^2-4x^2)z^2-2y^4-4x^2y^2-2x^4)$$

```
(%i151) ric1:ratsimp(subst(vx,diff(U,x),rics))$
(%i152) ric2:ratsimp(subst(vy,diff(U,y),ric1))$
(%i153) ric3:ratsimp(subst(vz,diff(U,z),ric2))$
(%i154) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$
(%i155) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$
(%i156) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$
(%i157) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$
```

```
(%i158) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$
```

```
(%i159) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$
```

```
(%i160) rica:ratsimp(subst(v0,U,ric9));
```

$$(\%o160)\ -(((v0^2-2\,v0+1)\,vzz+(v0-1)\,vyy-vy^2+(v0-1)\,vxx-vx^2)\,z^4+((2\,vy\,vz+(2-2\,v0)\,vyz)\,y+(2\,vx\,vz+(2-2\,v0)\,vxz)\,x+(2\,v0^2-6\,v0+4)\,vz)\,z^3+((2\,v0^2-3\,v0+1)\,vzz-vz^2+(-v0^2+2\,v0-1)\,vyy+(v0-1)\,vy^2+(v0-1)\,vxx-vx^2)\,y^2+((2\,v0\,vx\,vy+(2\,v0-2\,v0^2)\,vxy)\,x+(2-2\,v0)\,vy)\,y+((2\,v0^2-3\,v0+1)\,vzz-vz^2+(v0-1)\,vyy-vy^2+(-v0^2+2\,v0-1)\,vxx+(v0-1)\,vx^2)\,x^2+(2-2\,v0)\,vx\,x)\,z^2+(((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,y^3+(((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vz)\,y^2+((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,x^2\,y+((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x^3+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vz\,x^2)\,z+(v0-1)\,vz^2\,y^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy\,y^3+((2\,v0-2)\,vz^2\,x^2+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,y^2+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy\,x^2\,y+(v0-1)\,vz^2\,x^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x^3+(-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,x^2)\,/\,((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i161) ric1n:ratsimp(subst(vxn,vx,rica))$
```

```
(%i162) ric2n:ratsimp(subst(vyn,vy,ric1n))$
```

```
(%i163) ric3n:ratsimp(subst(vzn,vz,ric2n))$
```

```
(%i164) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$
```

```
(%i165) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$
```

```
(%i166) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$
```

```
(%i167) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$
```

```
(%i168) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$
```

```
(%i169) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$
```

```
(%i170) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i171) ratsimp(rican);
(%o171) 0
```

Ora verifico anche i termini NON DIAGONALI che possono essere diversi da zero se la funzione non possiede le opportune caratteristiche.

```
(%i172) rics:ratsimp(ric[2,3]);
```

$$(%o172)\ ((x\,y\,z^2\,U^2+(-2\,x\,y\,z^2-x\,y^3-x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_{z\,z})+(-x\,y\,z^2\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_z)^2+(((x\,z^3+(x^3-x\,y^2)\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)(U_y)+((y\,z^3+(y^3-x^2\,y)\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_x)-2\,x\,y\,z\,U^3+6\,x\,y\,z\,U^2-4\,x\,y\,z\,U)(U_z)+((-x\,y\,z^2-x^3\,y)\,U^2+(x\,y\,z^2+x^3\,y)\,U)(U_{y\,y})+(((x\,y^2-x^3)\,z-x\,z^3)\,U^2+(2\,x\,z^3+2\,x^3\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)(U_{y\,z})+(x\,y\,z^2+x^3\,y)\,U\,(U_y)^2+(((-z^4+(-y^2-x^2)\,z^2-2\,x^2\,y^2)\,U+z^4+(y^2+x^2)\,z^2)(U_x)-2\,x\,y^2\,U^3+(-x\,z^2+5\,x\,y^2-x^3)\,U^2+(2\,x\,z^2-2\,x\,y^2+2\,x^3)\,U-x\,z^2-x\,y^2-x^3)\,(U_y)+((-x\,y\,z^2-x\,y^3)\,U^2+(x\,y\,z^2+x\,y^3)\,U)(U_{x\,x})+(((x^2\,y-y^3)\,z-y\,z^3)\,U^2+(2\,y\,z^3+2\,y^3\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_{x\,z})+((-y^2-x^2)\,z^2-y^4-x^4)\,U^2+((2\,y^2+2\,x^2)\,z^2+2\,y^4+2\,x^2\,y^2+2\,x^4)\,U+(-y^2-x^2)\,z^2-y^4-2\,x^2\,y^2-x^4)\,(U_{x\,y})+(x\,y\,z^2+x\,y^3)\,U\,(U_x)^2+(-2\,x^2\,y\,U^3+(-y\,z^2-y^3+5\,x^2\,y)\,U^2+(2\,y\,z^2+2\,y^3-2\,x^2\,y)\,U-y\,z^2-y^3-x^2\,y)(U_x)-2\,x\,y\,U^4+6\,x\,y\,U^3-6\,x\,y\,U^2+2\,x\,y\,U)\,/\,((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^3+(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4)\,U^2+(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4)\,U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

```
(%i173) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i174) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i175) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i176) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i177) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i178) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i179) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i180) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$
```

```
(%i181) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$
```

```
(%i182) rica:ratsimp(subst(v0,U,ric9));
```

$$
\begin{aligned}
(\%o182)\ & -((v0-1)\,vx\,vy\,z^4+\\
&(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,y+((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x)\,z^3+(\\
&((v0-1)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y^2+(\\
&((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(v0^2-v0)\,vyy-v0\,vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+\\
&(v0^2-2\,v0+1)\,vx)\,y+((v0-1)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x^2+(v0^2-2\,v0+1)\,vy\,x)\,z^2\\
&+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,y^3+((v0+1)\,vy\,vz+(1-v0^2)\,vyz)\,x\,y^2+\\
&(((v0+1)\,vx\,vz+(1-v0^2)\,vxz)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vz\,x)\,y+\\
&((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^3)\,z+(v0^2-2\,v0+1)\,vxy\,y^4+\\
&(((v0-1)\,vzz-vz^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,y^3+\\
&((2\,v0\,vx\,vy+(2-2\,v0)\,vxy)\,x^2+(2\,v0^3-5\,v0^2+2\,v0+1)\,vy\,x)\,y^2+(\\
&((v0-1)\,vzz-vz^2+(v0^2-v0)\,vyy-v0\,vy^2)\,x^3+(2\,v0^3-5\,v0^2+2\,v0+1)\,vx\,x^2+\\
&(2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,x)\,y+(v0^2-2\,v0+1)\,vxy\,x^4+(v0^2-2\,v0+1)\,vy\,x^3)/(\\
&(2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+\\
&((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+\\
&(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\\
&x^4)
\end{aligned}
$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i183) ric1n:ratsimp(subst(vxn,vx,rica))$
```

```
(%i184) ric2n:ratsimp(subst(vyn,vy,ric1n))$
```

```
(%i185) ric3n:ratsimp(subst(vzn,vz,ric2n))$
```

```
(%i186) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$
```

```
(%i187) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$
```

```
(%i188) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$
```

```
(%i189) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$
```

```
(%i190) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$
```

```
(%i191) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$
```

```
(%i192) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i193) ratsimp(rican);
(%o193) 0
```

Verifico anche il termine NON DIAGONALE r[2,4] che puo' essere diverso da zero se la funzione non possiede le opportune caratteristiche.

```
(%i194) rics:ratsimp(ric[2,4]);
```

$$(\%o194)\ -(((x\,y^2+x^3)\,z\,U^2+(-x\,y^2-x^3)\,z\,U)(U_{z\,z})+(-x\,y^2-x^3)\,z\,U\,(U_z)^2+(((x\,y\,z^2-x\,y^3-x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_y)+(((y^2+2\,x^2)\,z^2+y^4+x^2\,y^2)\,U-y^2\,z^2-y^4-x^2\,y^2)(U_x)+2\,x\,z^2\,U^3+(-5\,x\,z^2+x\,y^2+x^3)\,U^2+(2\,x\,z^2-2\,x\,y^2-2\,x^3)\,U+x\,z^2+x\,y^2+x^3)\,(U_z)+(-x\,y^2\,z\,U^2+(x\,z^3+(2\,x\,y^2+x^3)\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)(U_{y\,y})+((-x\,y\,z^2+x\,y^3+x^3\,y)\,U^2+(-2\,x\,y^3-2\,x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_{y\,z})+(x\,y^2\,z\,U-x\,z^3+(-x\,y^2-x^3)\,z)(U_y)^2+\left(((x^2\,y-y^3)\,z-y\,z^3)\,U+y\,z^3+(y^3+x^2\,y)\,z)(U_x)+2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U\right)(U_y)+((x\,z^3+x\,y^2\,z)\,U^2+(-x\,z^3-x\,y^2\,z)\,U)(U_{x\,x})+((z^4+y^2\,z^2+x^2\,y^2+x^4)\,U^2+(-2\,z^4+(-2\,y^2-2\,x^2)\,z^2-2\,x^2\,y^2-2\,x^4)\,U+z^4+(y^2+2\,x^2)\,z^2+x^2\,y^2+x^4)\,(U_{x\,z})+((y\,z^3+(y^3-x^2\,y)\,z)\,U^2+(-2\,y\,z^3-2\,y^3\,z)\,U+y\,z^3+(y^3+x^2\,y)\,z)(U_{x\,y})+(-x\,z^3-x\,y^2\,z)\,U\,(U_x)^2+(2\,x^2\,z\,U^3+(z^3+(y^2-5\,x^2)\,z)\,U^2+((2\,x^2-2\,y^2)\,z-2\,z^3)\,U+z^3+(y^2+x^2)\,z)(U_x)+2\,x\,z\,U^4-6\,x\,z\,U^3+6\,x\,z\,U^2-2\,x\,z\,U)\,/\,((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^3+(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4)\,U^2+(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4)\,U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

```
(%i195) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i196) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i197) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i198) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i199) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i200) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i201) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$
```

```
(%i202) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$
```

```
(%i203) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$
```

```
(%i204) rica:ratsimp(subst(v0,U,ric9));
```

$$
\begin{aligned}
(\%o204)\ & -((v0^2-2\,v0+1)\,vxz\,z^4+(((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y+\\
& ((v0-1)\,vyy-vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,z^3+(\\
& ((v0-1)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,y^2+((v0+1)\,vy\,vz+(1-v0^2)\,vyz)\,x\,y+\\
& (2\,v0\,vx\,vz+(2-2\,v0)\,vxz)\,x^2+(2\,v0^3-5\,v0^2+2\,v0+1)\,vz\,x)\,z^2+(\\
& ((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y^3+(\\
& ((v0^2-v0)\,vzz-v0\,vz^2+(-v0^2+2\,v0-1)\,vyy+(v0-1)\,vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+\\
& (v0^2-2\,v0+1)\,vx)\,y^2+(((v0+1)\,vx\,vy+(1-v0^2)\,vxy)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vy\,x)\,y\\
& +((v0^2-v0)\,vzz-v0\,vz^2+(v0-1)\,vyy-vy^2)\,x^3+(2\,v0^3-5\,v0^2+2\,v0+1)\,vx\,x^2+\\
& (2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,x)\,z+(v0-1)\,vx\,vz\,y^4+\\
& ((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x\,y^3+\\
& (((v0-1)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x^2+(v0^2-2\,v0+1)\,vz\,x)\,y^2+\\
& ((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^3\,y+(v0^2-2\,v0+1)\,vxz\,x^4+(v0^2-2\,v0+1)\,vz\,x^3)\\
& /((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+\\
& ((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+\\
& (2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\\
& x^4)
\end{aligned}
$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i205) ric1n:ratsimp(subst(vxn,vx,rica))$
```

```
(%i206) ric2n:ratsimp(subst(vyn,vy,ric1n))$
```

```
(%i207) ric3n:ratsimp(subst(vzn,vz,ric2n))$
```

```
(%i208) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$
```

```
(%i209) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$
```

```
(%i210) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$
```

```
(%i211) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$
```

```
(%i212) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$
```

```
(%i213) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$
```

```
(%i214) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

(%i215) `ratsimp(rican);`

(%o215) 0

Ora verifico anche il termine NON DIAGONALE r[3,4] che potrebbe essere dive da zero se la funzione non possiede le opportune caratteristiche.

(%i216) `rics:ratsimp(ric[3,4]);`

(%o216)
$$-(((y^3+x^2\,y)\,z\,U^2+(-y^3-x^2\,y)\,z\,U)(U_{zz})+(-y^3-x^2\,y)\,z\,U\,(U_z)^2+(((2\,y^2+x^2)\,z^2+x^2\,y^2+x^4)\,U-x^2\,z^2-x^2\,y^2-x^4)(U_y)+((x\,y\,z^2-x\,y^3-x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_x)+2\,y\,z^2\,U^3+(-5\,y\,z^2+y^3+x^2\,y)\,U^2+(2\,y\,z^2-2\,y^3-2\,x^2\,y)\,U+y\,z^2+y^3+x^2\,y)\,(U_z)+((y\,z^3+x^2\,y\,z)\,U^2+(-y\,z^3-x^2\,y\,z)\,U)(U_{yy})+((z^4+x^2\,z^2+y^4+x^2\,y^2)\,U^2+(-2\,z^4+(-2\,y^2-2\,x^2)\,z^2-2\,y^4-2\,x^2\,y^2)\,U+z^4+(2\,y^2+x^2)\,z^2+y^4+x^2\,y^2)\,(U_{yz})+(-y\,z^3-x^2\,y\,z)\,U\,(U_y)^2+(((x\,y^2-x^3)\,z-x\,z^3)\,U+x\,z^3+(x\,y^2+x^3)\,z)(U_x)+2\,y^2\,z\,U^3+(z^3+(x^2-5\,y^2)\,z)\,U^2+((2\,y^2-2\,x^2)\,z-2\,z^3)\,U+z^3+(y^2+x^2)\,z)\,(U_y)+(-x^2\,y\,z\,U^2+(y\,z^3+(y^3+2\,x^2\,y)\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_{xx})+((-x\,y\,z^2+x\,y^3+x^3\,y)\,U^2+(-2\,x\,y^3-2\,x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)(U_{xz})+((x\,z^3+(x^3-x\,y^2)\,z)\,U^2+(-2\,x\,z^3-2\,x^3\,z)\,U+x\,z^3+(x\,y^2+x^3)\,z)(U_{xy})+(x^2\,y\,z\,U-y\,z^3+(-y^3-x^2\,y)\,z)(U_x)^2+(2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U)(U_x)+2\,y\,z\,U^4-6\,y\,z\,U^3+6\,y\,z\,U^2-2\,y\,z\,U)\,/\,((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^3+(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4)\,U^2+(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4)\,U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

(%i217) `ric1:ratsimp(subst(vx,diff(U,x),rics))$`

(%i218) `ric2:ratsimp(subst(vy,diff(U,y),ric1))$`

(%i219) `ric3:ratsimp(subst(vz,diff(U,z),ric2))$`

(%i220) `ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$`

(%i221) `ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$`

(%i222) `ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$`

(%i223) `ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$`

(%i224) `ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$`

(%i225)
```
ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$
```

(%i226)
```
rica:ratsimp(subst(v0,U,ric9));
```

(%o226)
$$-((v0^2-2\,v0+1)\,vyz\,z^4+(((v0^2-v0)\,vyy-v0\,vy^2+(v0-1)\,vxx-vx^2)\,y+((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x+(v0^2-2\,v0+1)\,vy)\,z^3+((2\,v0\,vy\,vz+(2-2\,v0)\,vyz)\,y^2+(((v0+1)\,vx\,vz+(1-v0^2)\,vxz)\,x+(2\,v0^3-5\,v0^2+2\,v0+1)\,vz)\,y+((v0-1)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^2)\,z^2+((((v0^2-v0)\,vzz-v0\,vz^2+(v0-1)\,vxx-vx^2)\,y^3+(((v0+1)\,vx\,vy+(1-v0^2)\,vxy)\,x+(2\,v0^3-5\,v0^2+2\,v0+1)\,vy)\,y^2+((((v0^2-v0)\,vzz-v0\,vz^2+(v0^2-v0)\,vyy-v0\,vy^2+(-v0^2+2\,v0-1)\,vxx+(v0-1)\,vx^2)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vx\,x+2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,y+((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x^3+(v0^2-2\,v0+1)\,vy\,x^2)\,z+(v0^2-2\,v0+1)\,vyz\,y^4+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x+(v0^2-2\,v0+1)\,vz)\,y^3+((v0-1)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^2\,y^2+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x^3+(v0^2-2\,v0+1)\,vz\,x^2)\,y+(v0-1)\,vy\,vz\,x^4)/((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i227)
```
ric1n:ratsimp(subst(vxn,vx,rica))$
```

(%i228)
```
ric2n:ratsimp(subst(vyn,vy,ric1n))$
```

(%i229)
```
ric3n:ratsimp(subst(vzn,vz,ric2n))$
```

(%i230)
```
ric4n:ratsimp(subst(vxxn,vxx,ric3n))$
```

(%i231)
```
ric5n:ratsimp(subst(vyyn,vyy,ric4n))$
```

(%i232)
```
ric6n:ratsimp(subst(vzzn,vzz,ric5n))$
```

(%i233)
```
ric7n:ratsimp(subst(vxyn,vxy,ric6n))$
```

(%i234)
```
ric8n:ratsimp(subst(vxzn,vxz,ric7n))$
```

(%i235)
```
ric9n:ratsimp(subst(vyzn,vyz,ric8n))$
```

(%i236)
```
rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i237) ratsimp(rican);
(%o237) 0
```

Per finire ristampo il tensore metrico covariante che ho usato per trovare la metrica del buco nero neutro in coordinate cartesiane.

```
(%i238) lg;
```

$$(\%o238)\quad \begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & -\dfrac{(z^2+y^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \dfrac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \dfrac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \dfrac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\dfrac{(z^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \dfrac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \dfrac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \dfrac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\dfrac{(y^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \end{bmatrix}$$

E per riepilogo ecco anche il tensore metrico in forma controvariante:

```
(%i239) ug;
```

$$(\%o239)\quad \begin{bmatrix} -\dfrac{1}{U-1} & 0 & 0 & 0 \\ 0 & \dfrac{x^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \dfrac{x\,y\,U}{z^2+y^2+x^2} & \dfrac{x\,z\,U}{z^2+y^2+x^2} \\ 0 & \dfrac{x\,y\,U}{z^2+y^2+x^2} & \dfrac{y^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \dfrac{y\,z\,U}{z^2+y^2+x^2} \\ 0 & \dfrac{x\,z\,U}{z^2+y^2+x^2} & \dfrac{y\,z\,U}{z^2+y^2+x^2} & \dfrac{z^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} \end{bmatrix}$$

Una caratteristica fondamentale di questa metrica è il fatto che il determinante vale -1 ossia è identico a quello della metrica pseudoeuclidea.

```
(%i240) ratsimp(determinant(lg));
(%o240) -1
```

```
(%i241) ratsimp(determinant(ug));
(%o241) -1
```