Modello di linearizzazione della RG

Giampaolo Bottoni - 11 aprile 2010

Propongo un modo innovativo per linearizzare (in modo NON lineare) le equazioni della Relatività Generale.
Con questo modello potrei studiare le interazioni gravitazionali tra buchi neri ed il pregio del modello è che, nel caso in cui uno solo dei buchi neri sia nettamente prevalente su tutti gli altri, le equazioni ottenute tendono ad essere quelle esatte ossia tali per cui il tensore di Ricci risulti ovunque nullo.
Nelle vicinanze di un singolo buco nero l'azione del buco nero è nettamente prevalente rispetto a quelle di tutti gli altri e dunque questo modello fornisce una ottima approssimazione delle equazioni della RG tanto migliore quanto più la RG si discosta dal classico modello gravitazionale newtoniano.

```
(%i1) (if atom(lg) then load(ctensor));
```
(%o1) C:/Programmi/Maxima-5.20.1/share/maxima/5.20.1/share/tensor/ctensor.mac

Inizializzo la libreria ctensor di Maxima.

```
(%i2) init_ctensor();
```
(%o2) done

Definisco una funzione del tutto generica delle coordinate oltre alla funzione di servizio rq.

```
(%i3) rq:x^2+y^2+z^2;
```
(%o3) $z^2+y^2+x^2$

La funzione U(x,y,z) del tutto generica è ricavata dalla classica funzione potenziale newtoniana. Specifico genericamente le variabili da cui dipende.

```
(%i4) depends(U,[x,y,z]);
```
(%o4) [U(x,y,z)]

La funzione b mi serve solo per semplificare l'input.

```
(%i5) b:U/(U-1);
```
(%o5) $\frac{U}{U-1}$

```
(%i6) lgmia: matrix (
[ 1-U, 0,0,0 ],
[ 0,-1+b*x^2/rq,b*x*y/rq, b*x*z/rq ],
[ 0,b*x*y/rq,-1+b*y^2/rq,b*y*z/rq ],
```

```
[ 0,b*x*z/rq,b*y*z/rq,-1+b*z^2/rq]);
```

(%o6)
$$\begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & \frac{x^2\,U}{(z^2+y^2+x^2)\,(U-1)}-1 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{x\,z\,U}{(z^2+y^2+x^2)\,(U-1)} \\ 0 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{y^2\,U}{(z^2+y^2+x^2)\,(U-1)}-1 & \frac{y\,z\,U}{(z^2+y^2+x^2)\,(U-1)} \\ 0 & \frac{x\,z\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,(U-1)} & \frac{z^2\,U}{(z^2+y^2+x^2)\,(U-1)}-1 \end{bmatrix}$$

Dichiaro alla libreria ctensor che faccio uso di coordinate cartesiane

```
(%i7) ct_coords: [t,x,y,z];
```

(%o7) $[t,x,y,z]$

In base alle esigenze della libreria ctensor di cui ho fatto il
load, inizializzo il tensore metrico covariante che si deve chiamare lg.

```
(%i8) lg:ratsimp(lgmia);
```

(%o8)
$$\begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & -\frac{(z^2+y^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\frac{(z^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \frac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\frac{(y^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \end{bmatrix}$$

Calcolo la metrica ovvero il tensore metrico controvariante
che deve essere la matrice inversa del tensore metrico
covariante.

```
(%i9) cmetric();
```

(%o9) *done*

```
(%i10) uug:ug$
```

Cerco di semplificare il piu' possibile l'espressione del
tensore metrico controvariante

```
(%i11) ug:ratsimp(uug);
```

(%o11)
$$\begin{bmatrix} -\frac{1}{U-1} & 0 & 0 & 0 \\ 0 & \frac{x^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \frac{x\,y\,U}{z^2+y^2+x^2} & \frac{x\,z\,U}{z^2+y^2+x^2} \\ 0 & \frac{x\,y\,U}{z^2+y^2+x^2} & \frac{y^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \frac{y\,z\,U}{z^2+y^2+x^2} \\ 0 & \frac{x\,z\,U}{z^2+y^2+x^2} & \frac{y\,z\,U}{z^2+y^2+x^2} & \frac{z^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} \end{bmatrix}$$

Anche se non serve ora faccio vedere che lg ed ug sono una la matrice inversa dell'altra

(%i12) ratsimp(ug . lg);

$$(\%o12)\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Ora calcolo i simboli di christoffel di prima e seconda specie visualizzandoli tutti. Attenzione alle regole della libreria ctensor che mette come terzo indice quello che di solito viene scritto come primo indice.

```
(%i13) christof(all);
```

(%t13) $lcs_{1,1,2}=\frac{U_x}{2}$

(%t14) $lcs_{1,1,3}=\frac{U_y}{2}$

(%t15) $lcs_{1,1,4}=\frac{U_z}{2}$

(%t16) $lcs_{1,2,1}=-\frac{U_x}{2}$

(%t17) $lcs_{1,3,1}=-\frac{U_y}{2}$

(%t18) $lcs_{1,4,1}=-\frac{U_z}{2}$

(%t19) $$lcs_{2,2,2}=\frac{\frac{((z^2+y^2)U-z^2-y^2-x^2)\left((z^2+y^2+x^2)(U_x)+2xU-2x\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{(z^2+y^2)(U_x)-2x}{(z^2+y^2+x^2)U-z^2-y^2-x^2}}{2}$$

(%t20) $lcs_{2,2,3}=$

$$\left(-\frac{((z^2+y^2)U-z^2-y^2-x^2)\left((z^2+y^2+x^2)(U_y)+2yU-2y\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{(z^2+y^2)(U_y)+2yU-2y}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right.$$
$$\left.-\frac{2xyU\left((z^2+y^2+x^2)(U_x)+2xU-2x\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{2xy(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{2yU}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$ (%t21)

$$\left(-\frac{((z^2+y^2)U-z^2-y^2-x^2)\left((z^2+y^2+x^2)(U_z)+2zU-2z\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{(z^2+y^2)(U_z)+2zU-2z}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right.$$
$$\left.-\frac{2xzU\left((z^2+y^2+x^2)(U_x)+2xU-2x\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{2xz(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{2zU}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$ (%t22)

(%t23) $$lcs_{2,3,3}=\frac{\frac{((z^2+x^2)U-z^2-y^2-x^2)\left((z^2+y^2+x^2)(U_x)+2xU-2x\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{(z^2+x^2)(U_x)+2xU-2x}{(z^2+y^2+x^2)U-z^2-y^2-x^2}}{2}$$

(%t24) $lcs_{2,3,4}=$

$$\left(\frac{xyU\left((z^2+y^2+x^2)(U_z)+2zU-2z\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-\frac{xy(U_z)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\right.$$
$$\frac{xzU\left((z^2+y^2+x^2)(U_y)+2yU-2y\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+\frac{xz(U_y)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{yzU\left((z^2+y^2+x^2)(U_x)+2xU-2x\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}+$$
$$\left.\frac{yz(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$ (%t25) $$lcs_{2,4,2}=\frac{\frac{((z^2+y^2+x^2)U-z^2-y^2-x^2)\left((z^2+y^2+x^2)(U_y)+2zU-2z\right)}{((z^2+y^2+x^2)U-z^2-y^2-x^2)^2}-}{2}$$

(%t26) $lcs_{2,4,3}=$

$$\left(-\frac{x\,y\,U\left((z^2+y^2+x^2)(U_z)+2\,z\,U-2\,z\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{x\,y(U_z)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{x\,z\,U\left((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}-\frac{x\,z(U_y)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{y\,z\,U\left((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{y\,z(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$

(%t27) $lcs_{2,4,4}=\dfrac{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}{2}$

(%t28) $lcs_{3,3,2}=$

$$\left(-\frac{2\,x\,y\,U\left((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{2\,x\,y(U_y)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{\left((z^2+x^2)U-z^2-y^2-x^2\right)\left((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{(z^2+x^2)(U_x)+2\,x\,U-2\,x}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{\left((z^2+x^2)U-z^2-y^2-x^2\right)\left((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}-\frac{2\,x\,U}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$

(%t29) $lcs_{3,3,3}=\dfrac{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}{2}$

(%t30) $lcs_{3,3,4}=$

$$\left(-\frac{\left((z^2+x^2)U-z^2-y^2-x^2\right)\left((z^2+y^2+x^2)(U_z)+2\,z\,U-2\,z\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{(z^2+x^2)(U_z)+2\,z\,U-2\,z}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{2\,y\,z\,U\left((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{2\,y\,z(U_y)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{2\,z\,U}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$

(%t31

$$\left(-\frac{x\,y\,U\left((z^2+y^2+x^2)(U_z)+2\,z\,U-2\,z\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{x\,y(U_z)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}-\frac{x\,z\,U\left((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}+\frac{x\,z(U_y)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}+\frac{y\,z\,U\left((z^2+y^2+x^2)(U_x)+2\,x\,U-2\,x\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}-\frac{y\,z(U_x)}{(z^2+y^2+x^2)U-z^2-y^2-x^2}\right)/2$$

(%t32) $lcs_{3,4,3}=\dfrac{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}{2}$

(%t33) $lcs_{3,4,4}=\dfrac{\left((y^2+x^2)U-z^2-y^2-x^2\right)\left((z^2+y^2+x^2)(U_y)+2\,y\,U-2\,y\right)}{\left((z^2+y^2+x^2)U-z^2-y^2-x^2\right)^2}-\dfrac{(y^2+x^2)(U_y)+2\,y\,U-2\,y}{(z^2+y^2+x^2)U-z^2-y^2-x^2}}{2}$

(%t34) $lcs_{4,4,2}=$

$$\left(-\frac{2\,x\,z\,U\left(\left(z^2+y^2+x^2\right)\left(U_z\right)+2\,z\,U-2\,z\right)}{\left(\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2\right)^2}+\frac{2\,x\,z\left(U_z\right)}{\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2}-\right.$$
$$\frac{\left(\left(y^2+x^2\right)U-z^2-y^2-x^2\right)\left(\left(z^2+y^2+x^2\right)\left(U_x\right)+2\,x\,U-2\,x\right)}{\left(\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2\right)^2}+\frac{\left(y^2+x^2\right)\left(U_x\right)+2\,x\,U-2\,x}{\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2}+$$
$$\left.\frac{2\,x\,U}{\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2}\right)/2 \quad (\%t35)\quad lcs_{4,4,3}=$$

$$\left(-\frac{2\,y\,z\,U\left(\left(z^2+y^2+x^2\right)\left(U_z\right)+2\,z\,U-2\,z\right)}{\left(\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2\right)^2}+\frac{2\,y\,z\left(U_z\right)}{\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2}-\right.$$
$$\frac{\left(\left(y^2+x^2\right)U-z^2-y^2-x^2\right)\left(\left(z^2+y^2+x^2\right)\left(U_y\right)+2\,y\,U-2\,y\right)}{\left(\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2\right)^2}+\frac{\left(y^2+x^2\right)\left(U_y\right)+2\,y\,U-2\,y}{\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2}+$$
$$\left.\frac{2\,y\,U}{\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2}\right)/2 \quad (\%t36)\quad lcs_{4,4,4}=\frac{\left(\left(y^2+x^2\right)U-z^2-y^2-x^2\right)\left(\left(z^2+y^2+x^2\right)\left(U_z\right)+2\,z\,U-2\,z\right)}{\left(\left(z^2+y^2+x^2\right)U-z^2-y^2-x^2\right)^2}}{2}$$

$$(\%t37)\quad mcs_{1,1,2}=\frac{x\,z\,U\left(U_z\right)+x\,y\,U\left(U_y\right)+\left(x^2\,U-z^2-y^2-x^2\right)\left(U_x\right)}{2\,z^2+2\,y^2+2\,x^2}$$

$$(\%t38)\quad mcs_{1,1,3}=\frac{y\,z\,U\left(U_z\right)+\left(y^2\,U-z^2-y^2-x^2\right)\left(U_y\right)+x\,y\,U\left(U_x\right)}{2\,z^2+2\,y^2+2\,x^2}$$

$$(\%t39)\quad mcs_{1,1,4}=\frac{\left(z^2\,U-z^2-y^2-x^2\right)\left(U_z\right)+y\,z\,U\left(U_y\right)+x\,z\,U\left(U_x\right)}{2\,z^2+2\,y^2+2\,x^2}$$

$$(\%t40)\quad mcs_{1,2,1}=\frac{U_x}{2\,U-2}$$

$$(\%t41)\quad mcs_{1,3,1}=\frac{U_y}{2\,U-2}$$

$$(\%t42)\quad mcs_{1,4,1}=\frac{U_z}{2\,U-2}$$

$$(\%t43)\quad mcs_{2,2,2}=$$
$$\left(x^3\,z\,U\left(U_z\right)+x^3\,y\,U\left(U_y\right)+\left(\left(-2\,x^2\,z^2-2\,x^2\,y^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_x\right)+\right.$$
$$\left.\left(2\,x\,z^2+2\,x\,y^2\right)U^3+\left(-4\,x\,z^2-4\,x\,y^2\right)U^2+\left(2\,x\,z^2+2\,x\,y^2\right)U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\right.\right.$$
$$\left.\left.2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right) \quad (\%t44)\quad mcs_{2,2,3}=\right.$$
$$\left(x^2\,y\,z\,U\left(U_z\right)+\left(x^2\,y^2\,U-x^2\,z^2-x^2\,y^2-x^4\right)\left(U_y\right)+\right.$$
$$\left.\left(\left(-2\,x\,y\,z^2-2\,x\,y^3-x^3\,y\right)U+2\,x\,y\,z^2+2\,x\,y^3+2\,x^3\,y\right)\left(U_x\right)+\left(2\,y\,z^2+2\,y^3\right)U^3+\left(-4\,y\,z^2-4\,y^3\right)U^2\right.$$
$$\left.+\left(2\,y\,z^2+2\,y^3\right)U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+\right.$$
$$\left.\left(-4\,z^4+\left(-8\,y^2-8\,x^2\right)z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right) \quad (\%t$$
$$\left(\left(x^2\,z^2\,U-x^2\,z^2-x^2\,y^2-x^4\right)\left(U_z\right)+x^2\,y\,z\,U\left(U_y\right)+\right.$$
$$\left.\left(\left(\left(-2\,x\,y^2-x^3\right)z-2\,x\,z^3\right)U+2\,x\,z^3+\left(2\,x\,y^2+2\,x^3\right)z\right)\left(U_x\right)+\left(2\,z^3+2\,y^2\,z\right)U^3+\left(-4\,z^3-4\,y^2\,z\right)U^2+\right.$$
$$\left.\left(2\,z^3+2\,y^2\,z\right)U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+\right.$$
$$\left.\left(-4\,z^4+\left(-8\,y^2-8\,x^2\right)z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right) \quad (\%t$$
$$\left(x^2\,y\,z\,U\left(U_z\right)+\left(\left(-x^2\,z^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_y\right)+\left(-x\,y\,z^2-x\,y^3\right)U\left(U_x\right)\right.$$

$$-2\,x^2\,y\,U^3+4\,x^2\,y\,U^2-2\,x^2\,y\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(x\,y^2\,z\,U\,(U_z)+(-x\,y\,z^2-x^3\,y)\,U\,(U_y)+\left((-y^2\,z^2-y^4)\,U+y^2\,z^2+y^4+x^2\,y^2\right)$$
$$(U_x)-2\,x\,y^2\,U^3+4\,x\,y^2\,U^2-2\,x\,y^2\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$((x\,y\,z^2\,U-x\,y\,z^2-x\,y^3-x^3\,y)\,(U_z)+\left((-x\,z^3-x^3\,z)\,U+x\,z^3+(x\,y^2+x^3)\,z\right)(U_y)$$
$$+\left((-y\,z^3-y^3\,z)\,U+y\,z^3+(y^3+x^2\,y)\,z\right)(U_x)-2\,x\,y\,z\,U^3+4\,x\,y\,z\,U^2-2\,x\,y\,z\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t49)\quad mcs_{2,4,2}=-$$
$$(\left((x^2\,y^2+x^4)\,U-x^2\,z^2-x^2\,y^2-x^4\right)(U_z)-x^2\,y\,z\,U\,(U_y)+(x\,z^3+x\,y^2\,z)\,U\,(U_x)+$$
$$2\,x^2\,z\,U^3-4\,x^2\,z\,U^2+2\,x^2\,z\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(\left((x\,y^3+x^3\,y)\,U-x\,y\,z^2-x\,y^3-x^3\,y\right)(U_z)+\left(-x\,y^2\,z\,U+x\,z^3+(x\,y^2+x^3)\,z\right)$$
$$(U_y)+\left((y\,z^3+y^3\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z\right)(U_x)+2\,x\,y\,z\,U^3-4\,x\,y\,z\,U^2+2\,x\,y\,z\,U)\,/\,(\left(2\,z^4+(4\,y^2\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t51)\quad mcs_{2,4,4}=-$$
$$((x\,y^2+x^3)\,z\,U\,(U_z)-x\,y\,z^2\,U\,(U_y)+\left((z^4+y^2\,z^2)\,U-z^4+(-y^2-x^2)\,z^2\right)(U_x)+2$$
$$x\,z^2\,U^3-4\,x\,z^2\,U^2+2\,x\,z^2\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(x\,y^2\,z\,U\,(U_z)+\left((-2\,x\,y\,z^2-x\,y^3-2\,x^3\,y)\,U+2\,x\,y\,z^2+2\,x\,y^3+2\,x^3\,y\right)(U_y)+$$
$$(x^2\,y^2\,U-y^2\,z^2-y^4-x^2\,y^2)\,(U_x)+(2\,x\,z^2+2\,x^3)\,U^3+(-4\,x\,z^2-4\,x^3)\,U^2+(2\,x\,z^2+2\,x^3)\,U)\,/\,(\left(2\,z^4+\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t53)\quad mcs_{3,3,3}=$$
$$(y^3\,z\,U\,(U_z)+\left((-2\,y^2\,z^2-y^4-2\,x^2\,y^2)\,U+y^2\,z^2+y^4+x^2\,y^2\right)(U_y)+x\,y^3\,U\,(U_x)+$$
$$(2\,y\,z^2+2\,x^2\,y)\,U^3+(-4\,y\,z^2-4\,x^2\,y)\,U^2+(2\,y\,z^2+2\,x^2\,y)\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t54)\quad mcs_{3,3,4}=$$
$$((y^2\,z^2\,U-y^2\,z^2-y^4-x^2\,y^2)\,(U_z)+$$
$$\left(\left((-y^3-2\,x^2\,y)\,z-2\,y\,z^3\right)U+2\,y\,z^3+(2\,y^3+2\,x^2\,y)\,z\right)(U_y)+x\,y^2\,z\,U\,(U_x)+(2\,z^3+2\,x^2\,z)\,U^3+$$
$$(-4\,z^3-4\,x^2\,z)\,U^2+(2\,z^3+2\,x^2\,z)\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(\left((x\,y^3+x^3\,y)\,U-x\,y\,z^2-x\,y^3-x^3\,y\right)(U_z)+$$
$$\left((x\,z^3+x^3\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z\right)(U_y)+\left(-x^2\,y\,z\,U+y\,z^3+(y^3+x^2\,y)\,z\right)(U_x)+2\,x\,y\,z\,U^3-4\,x\,y\,z$$
$$U^2+2\,x\,y\,z\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(\left((y^4+x^2\,y^2)\,U-y^2\,z^2-y^4-x^2\,y^2\right)(U_z)+(y\,z^3+x^2\,y\,z)\,U\,(U_y)-x\,y^2\,z\,U\,(U_x)$$
$$+2\,y^2\,z\,U^3-4\,y^2\,z\,U^2+2\,y^2\,z\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$((y^3+x^2\,y)\,z\,U\,(U_z)+\left((z^4+x^2\,z^2)\,U-z^4+(-y^2-x^2)\,z^2\right)(U_y)-x\,y\,z^2\,U\,(U_x)+2$$
$$x^2\,z\,U^3-4\,x^2\,z\,U^2+2\,x^2\,z\,U)\,/\,(\left(2\,z^4+(4\,x^2+4\,x^2)\,z^2+2\,x^4+4\,x^2\,x^2+2\,x^4\right)x^2+$$

$$y\,z^2\,U^2-4\,y\,z^2\,U^2+2\,y\,z^2\,U)/((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(\left(\left(x\,z^3+(2\,x\,y^2+2\,x^3)\,z\right)U-2\,x\,z^3+(-2\,x\,y^2-2\,x^3)\,z\right)(U_z)-x\,y\,z^2\,U\,(U_y)+$$
$$\left(-x^2\,z^2\,U+z^4+(y^2+x^2)\,z^2\right)(U_x)+(-2\,x\,y^2-2\,x^3)\,U^3+(4\,x\,y^2+4\,x^3)\,U^2+(-2\,x\,y^2-2\,x^3)\,U)/(\left(2\,z^4\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t59)\quad mcs_{4,4,3}=-$$
$$(\left(\left(y\,z^3+(2\,y^3+2\,x^2\,y)\,z\right)U-2\,y\,z^3+(-2\,y^3-2\,x^2\,y)\,z\right)(U_z)+$$
$$\left(-y^2\,z^2\,U+z^4+(y^2+x^2)\,z^2\right)(U_y)-x\,y\,z^2\,U\,(U_x)+(-2\,y^3-2\,x^2\,y)\,U^3+(4\,y^3+4\,x^2\,y)\,U^2+$$
$$(-2\,y^3-2\,x^2\,y)\,U)/(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^2+$$
$$\left(-4\,z^4+(-8\,y^2-8\,x^2)\,z^2-4\,y^4-8\,x^2\,y^2-4\,x^4\right)U+2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%t$$
$$(\left(\left(z^4+(2\,y^2+2\,x^2)\,z^2\right)U-z^4+(-y^2-x^2)\,z^2\right)(U_z)-y\,z^3\,U\,(U_y)-x\,z^3\,U\,(U_x)+$$
$$(-2\,y^2-2\,x^2)\,z\,U^3+(4\,y^2+4\,x^2)\,z\,U^2+(-2\,y^2-2\,x^2)\,z\,U)/(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\quad(\%o60)\quad done$$

Provo a semplificare una delle componenti del simbolo di Christoffel
di seconda specie.

```
(%i61) ratsimp(mcs[2,2,2]);
```

$$(\%o61)\quad (x^3\,z\,U\,(U_z)+x^3\,y\,U\,(U_y)+\left(\left(-2\,x^2\,z^2-2\,x^2\,y^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)(U_x)+$$
$$(2\,x\,z^2+2\,x\,y^2)\,U^3+(-4\,x\,z^2-4\,x\,y^2)\,U^2+(2\,x\,z^2+2\,x\,y^2)\,U)/(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\right.$$
$$2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)$$

Creo il tensore di Riemann sopprimendo l'output che probabilmente sarebbe
troppo lungo per essere stampato.

```
(%i62) riemann(false);
```

$$(\%o62)\quad done$$

Ora faccio lo stesso calcolando il tensore di Ricci.

```
(%i63) ricci(false);
```

$$(\%o63)\quad done$$

Ora trasformo il tensore di Ricci in matrice.

```
(%i64) mat_ricci: ratsimp( matrix(
[ ric[1,1],ric[1,2],ric[1,3],ric[1,4]],
[ ric[2,1],ric[2,2],ric[2,3],ric[2,4]],
[ ric[3,1],ric[3,2],ric[3,3],ric[3,4]],
[ ric[4,1],ric[4,2],ric[4,3],ric[4,4]]))$
```

Innanzi tutto controllo che i termini non diagonali della
prima riga e colonna siano degli zeri.

```
(%i65) forsezeri: [ratsimp(ric[2,1]),ratsimp(ric[3,1]),ratsimp(ric[4,1]),
ratsimp(ric[1,2]),
ratsimp(ric[1,3]),
ratsimp(ric[1,4])];
```

(%o65) [0,0,0,0,0,0]

Controllo che il tensore di ricci e' un tensore simmetrico... come da manuale
ma se uno vuol fare il san Tommaso...

```
(%i66) certozeri: [ratsimp(ric[2,3]-ric[3,2]), ratsimp(ric[2,4]-ric[4,2]),
ratsimp(ric[3,4]-ric[4,3]) ];
```

(%o66) [0,0,0]

Questi non sono zeri ma se la funzione U(x,y,z) assume
dei valori adeguati lo debbono diventare...
Intanto li stampo...

```
(%i67) ratsimp(ric[1,1]);
```

(%o67) $$(\left(z^2\,U^2+\left(-2\,z^2-y^2-x^2\right)U+z^2+y^2+x^2\right)\left(U_{z\,z}\right)+\left(y^2+x^2\right)\left(U_z\right)^2+\left(-2\,y\,z\left(U_y\right)-2\,x\,z\left(U_x\right)+2\,z\,U^2-2\,z\,U\right)\left(U_z\right)+\left(y^2\,U^2+\left(-z^2-2\,y^2-x^2\right)U+z^2+y^2+x^2\right)\left(U_{y\,y}\right)+\left(2\,y\,z\,U^2-2\,y\,z\,U\right)\left(U_{y\,z}\right)+\left(z^2+x^2\right)\left(U_y\right)^2+\left(-2\,x\,y\left(U_x\right)+2\,y\,U^2-2\,y\,U\right)\left(U_y\right)+\left(x^2\,U^2+\left(-z^2-y^2-2\,x^2\right)U+z^2+y^2+x^2\right)\left(U_{x\,x}\right)+\left(2\,x\,z\,U^2-2\,x\,z\,U\right)\left(U_{x\,z}\right)+\left(2\,x\,y\,U^2-2\,x\,y\,U\right)\left(U_{x\,y}\right)+\left(z^2+y^2\right)\left(U_x\right)^2+\left(2\,x\,U^2-2\,x\,U\right)\left(U_x\right))\,/\,\left(\left(2\,z^2+2\,y^2+2\,x^2\right)U-2\,z^2-2\,y^2-2\,x^2\right)$$

```
(%i68) ratsimp(ric[2,2]);
```

(%o68) $$(\left(x^2\,z^2\,U^2+\left(-2\,x^2\,z^2-x^2\,y^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_{z\,z}\right)+\left(-x^2\,z^2\,U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_z\right)^2+\left(-2\,x^2\,y\,z\,U\left(U_y\right)+\left(\left(2\,x\,z^3+2\,x\,y^2\,z\right)U-2\,x\,z^3+\left(-2\,x\,y^2-2\,x^3\right)z\right)\left(U_x\right)+\left(2\,z^3+2\,y^2\,z\right)U^3+\left(-6\,z^3-6\,y^2\,z\right)U^2+\left(6\,z^3+\left(6\,y^2+2\,x^2\right)z\right)U-2\,z^3+\left(-2\,y^2-2\,x^2\right)z\right)\left(U_z\right)+\left(x^2\,y^2\,U^2+\left(-x^2\,z^2-2\,x^2\,y^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_{y\,y}\right)+\left(2\,x^2\,y\,z\,U^2-2\,x^2\,y\,z\,U\right)\left(U_{y\,z}\right)+\left(-x^2\,y^2\,U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_y\right)^2+\left(\left(\left(2\,x\,y\,z^2+2\,x\,y^3\right)U-2\,x\,y\,z^2-2\,x\,y^3-2\,x^3\,y\right)\left(U_x\right)+\left(2\,y\,z^2+2\,y^3\right)U^3+\left(-6\,y\,z^2-6\,y^3\right)U^2+\left(6\,y\,z^2+6\,y^3+2\,x^2\,y\right)U-2\,y\,z^2-2\,y^3-2\,x^2\,y\right)\left(U_y\right)+\left(\left(-2\,x^2\,z^2-2\,x^2\,y^2-x^4\right)U^2+\left(3\,x^2\,z^2+3\,x^2\,y^2+2\,x^4\right)U-x^2\,z^2-x^2\,y^2-x^4\right)\left(U_{x\,x}\right)+\left(\left(-2\,x\,z^3-2\,x\,y^2\,z\right)U^2+\left(4\,x\,z^3+\left(4\,x\,y^2+2\,x^3\right)z\right)U-2\,x\,z^3+\left(-2\,x\,y^2-2\,x^3\right)z\right)\left(U_{x\,z}\right)+\left(\left(-2\,x\,y\,z^2-2\,x\,y^3\right)U^2+\left(4\,x\,y\,z^2+4\,x\,y^3+2\,x^3\,y\right)U-2\,x\,y\,z^2-2\,x\,y^3-2\,x^3\,y\right)\left(U_{x\,y}\right)+\left(\left(-z^4-2\,y^2\,z^2-y^4\right)U+z^4+\left(2\,y^2+x^2\right)z^2+y^4+x^2\,y^2\right)\left(U_x\right)^2+\left(\left(2\,x\,z^2+2\,x\,y^2\right)U^3+\left(-8\,x\,z^2-8\,x\,y^2-2\,x^3\right)U^2+\left(10\,x\,z^2+10\,x\,y^2+6\,x^3\right)U-4\,x\,z^2-4\,x\,y^2-4\,x^3\right)\left(U_x\right)+\left(2\,z^2+2\,y^2\right)U^4+\left(-6\,z^2-6\,y^2\right)U^3+\left(6\,z^2+6\,y^2\right)U^2+\left(-2\,z^2-2\,y^2\right)U)\,/\,(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2\right.$$

$$\left.U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right.$$

(%i69) ratsimp(ric[3,3]);

$$(\%o69)\quad -\left(\left(y^2\,z^2\,U^2+\left(-2\,y^2\,z^2-y^4-x^2\,y^2\right)U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_{z\,z}\right)+\left(-y^2\,z^2\,U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_z\right)^2+\left(\left(\left(2\,y\,z^3+2\,x^2\,y\,z\right)U-2\,y\,z^3+\left(-2\,y^3-2\,x^2\,y\right)z\right)\left(U_y\right)-2\,x\,y^2\,z\,U\left(U_x\right)+\left(2\,z^3+2\,x^2\,z\right)U^3+\left(-6\,z^3-6\,x^2\,z\right)U^2+\left(6\,z^3+\left(2\,y^2+6\,x^2\right)z\right)U-2\,z^3+\left(-2\,y^2-2\,x^2\right)z\right)\left(U_z\right)+\left(\left(-2\,y^2\,z^2-y^4-2\,x^2\,y^2\right)U^2+\left(3\,y^2\,z^2+2\,y^4+3\,x^2\,y^2\right)U-y^2\,z^2-y^4-x^2\,y^2\right)\left(U_{y\,y}\right)+\left(\left(-2\,y\,z^3-2\,x^2\,y\,z\right)U^2+\left(4\,y\,z^3+\left(2\,y^3+4\,x^2\,y\right)z\right)U-2\,y\,z^3+\left(-2\,y^3-2\,x^2\,y\right)z\right)\left(U_{y\,z}\right)+\left(\left(-z^4-2\,x^2\,z^2-x^4\right)U+z^4+\left(y^2+2\,x^2\right)z^2+x^2\,y^2+x^4\right)\left(U_y\right)^2+\left(\left(\left(2\,x\,y\,z^2+2\,x^3\,y\right)U-2\,x\,y\,z^2-2\,x\,y^3\right)U^2+\left(10\,y\,z^2+6\,y^3+10\,x^2\,y\right)U-4\,y\,z^2-4\,y^3-4\,x^2\,y\right)\left(U_y\right)+\left(x^2\,y^2\,U^2+\left(-y^2\,z^2-y^4-2\,x^2\,y^2\right)U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_{x\,x}\right)+\left(2\,x\,y^2\,z\,U^2-2\,x\,y^2\,z\,U\right)\left(U_{x\,z}\right)+\left(\left(-2\,x\,y\,z^2-2\,x^3\,y\right)U^2+\left(4\,x\,y\,z^2+2\,x\,y^3+4\,x^3\,y\right)U-2\,x\,y\,z^2-2\,x\,y^3-2\,x^3\,y\right)\left(U_{x\,y}\right)+\left(-x^2\,y^2\,U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_x\right)^2+\left(\left(2\,x\,z^2+2\,x^3\right)U^3+\left(-6\,x\,z^2-6\,x^3\right)U^2+\left(6\,x\,z^2+2\,x\,y^2+6\,x^3\right)U-2\,x\,z^2-2\,x\,y^2-2\,x^3\right)\left(U_x\right)+\left(2\,z^2+2\,x^2\right)U^4+\left(-6\,z^2-6\,x^2\right)U^3+\left(6\,z^2+6\,x^2\right)U^2+\left(-2\,z^2-2\,x^2\right)U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4\right)U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right.$$

(%i70) ratsimp(ric[4,4]);

$$(\%o70)\quad -\left(\left(\left(z^4+\left(2\,y^2+2\,x^2\right)z^2\right)U^2+\left(\left(-3\,y^2-3\,x^2\right)z^2-2\,z^4\right)U+z^4+\left(y^2+x^2\right)z^2\right)\left(U_{z\,z}\right)+\left(\left(y^4+2\,x^2\,y^2+x^4\right)U+\left(-y^2-x^2\right)z^2-y^4-2\,x^2\,y^2-x^4\right)\left(U_z\right)^2+\left(\left(\left(-2\,y^3-2\,x^2\,y\right)z\,U+2\,y\,z^3+\left(2\,y^3+2\,x^2\,y\right)z\right)\left(U_x\right)+\left(-2\,y^2-2\,x^2\right)z\,U^3+\left(2\,z^3+\left(8\,y^2+8\,x^2\right)z\right)U^2+\left(\left(-10\,y^2-10\,x^2\right)z-6\,z^3\right)U+4\,z^3+\left(4\,y^2+4\,x^2\right)z\right)\left(U_z\right)+\left(-y^2\,z^2\,U^2+\left(z^4+\left(2\,y^2+x^2\right)z^2\right)U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_{y\,y}\right)+\left(\left(2\,y^3+2\,x^2\,y\right)z\,U^2+\left(\left(-4\,y^3-4\,x^2\,y\right)z-2\,y\,z^3\right)U+2\,y\,z^3+\left(2\,y^3+2\,x^2\,y\right)z\right)\left(U_{y\,z}\right)+\left(y^2\,z^2\,U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_y\right)^2+\left(2\,x\,y\,z^2\,U\left(U_x\right)+\left(-2\,y^3-2\,x^2\,y\right)U^3+\left(6\,y^3+6\,x^2\,y\right)U^2+\left(-2\,y\,z^2-6\,y^3-6\,x^2\,y\right)U+2\,y\,z^2+2\,y^3+2\,x^2\,y\right)\left(U_y\right)+\left(-x^2\,z^2\,U^2+\left(z^4+\left(y^2+2\,x^2\right)z^2\right)U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_{x\,x}\right)+\left(\left(2\,x\,y^2+2\,x^3\right)z\,U^2+\left(\left(-4\,x\,y^2-4\,x^3\right)z-2\,x\,z^3\right)U+2\,x\,z^3+\left(2\,x\,y^2+2\,x^3\right)z\right)\left(U_{x\,z}\right)+\left(2\,x\,y\,z^2\,U-2\,x\,y\,z^2\,U^2\right)\left(U_{x\,y}\right)+\left(x^2\,z^2\,U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_x\right)^2+\left(\left(-2\,x\,y^2-2\,x^3\right)U^3+\left(6\,x\,y^2+6\,x^3\right)U^2+\left(-2\,x\,z^2-6\,x\,y^2-6\,x^3\right)U+2\,x\,z^2+2\,x\,y^2+2\,x^3\right)\left(U_x\right)+\left(-2\,y^2-2\,x^2\right)U^4+\left(6\,y^2+6\,x^2\right)U^3+\left(-6\,y^2-6\,x^2\right)U^2+\left(2\,y^2+2\,x^2\right)U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4\right)U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right.$$

(%i71) ratsimp(ric[2,3]);

(%o71) $$\left(x\,y\,z^2\,U^2+\left(-2\,x\,y\,z^2-x\,y^3-x^3\,y\right)U+x\,y\,z^2+x\,y^3+x^3\,y\right)\left(U_{z\,z}\right)+\left(-x\,y\,z^2\,U+x\,y\,z^2+x\,y^3+x^3\,y\right)\left(U_z\right)^2+\left(\left(\left(x\,z^3+\left(x^3-x\,y^2\right)z\right)U-x\,z^3+\left(-x\,y^2-x^3\right)z\right)\left(U_y\right)+\left(\left(y\,z^3+\left(y^3-x^2\,y\right)z\right)U-y\,z^3+\left(-y^3-x^2\,y\right)z\right)\left(U_x\right)-2\,x\,y\,z\,U^3+6\,x\,y\,z\,U^2-4\,x\,y\,z\,U\right)\left(U_z\right)+\left(\left(-x\,y\,z^2-x^3\,y\right)U^2+\left(x\,y\,z^2+x^3\,y\right)U\right)\left(U_{y\,y}\right)+\left(\left(\left(x\,y^2-x^3\right)z-x\,z^3\right)U^2+\left(2\,x\,z^3+2\,x^3\,z\right)U-x\,z^3+\left(-x\,y^2-x^3\right)z\right)\left(U_{y\,z}\right)+\left(x\,y\,z^2+x^3\,y\right)U\left(U_y\right)^2+\left(\left(-\left(2\,x\,z^2-2\,x\,y^2+2\,x^3\right)U-x\,z^2-x\,y^2-x^3\right)\left(U_y\right)+\left(\left(-x\,y\,z^2-x\,y^3\right)U^2+\left(x\,y\,z^2+x\,y^3\right)U\right)\left(U_{x\,x}\right)+\left(\left(\left(x^2\,y-y^3\right)z-y\,z^3\right)U^2+\left(2\,y\,z^3+2\,y^3\,z\right)U-y\,z^3+\left(-y^3-x^2\,y\right)z\right)\left(U_{x\,z}\right)+\left(\left(\left(-y^2-x^2\right)z^2-y^4-x^4\right)U^2+\left(\left(2\,y^2+2\,x^2\right)z^2+2\,y^4+2\,x^2\,y^2+2\,x^4\right)U+\left(-y^2-x^2\right)z^2-y^4-2\,x^2\,y^2-x^4\right)\left(U_{x\,y}\right)+\left(x\,y\,z^2+x\,y^3\right)U\left(U_x\right)^2+\left(-2\,x^2\,y\,U^3+\left(-y\,z^2-y^3+5\,x^2\,y\right)U^2+\left(2\,y\,z^2+2\,y^3-2\,x^2\,y\right)U-y\,z^2-y^3-x^2\,y\right)\left(U_x\right)-2\,x\,y\,U^4+6\,x\,y\,U^3-6\,x\,y\,U^2+2\,x\,y\,U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^3+\left(-6\,z^4+\left(-12\,y^2-12\,x^2\right)z^2-6\,y^4-12\,x^2\,y^2-6\,x^4\right)U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right)$$

(%i72) ratsimp(ric[2,4]);

(%o72) $$-\left(\left(x\,y^2+x^3\right)z\,U^2+\left(-x\,y^2-x^3\right)z\,U\right)\left(U_{z\,z}\right)+\left(-x\,y^2-x^3\right)z\,U\left(U_z\right)^2+\left(\left(x\,y\,z^2-x\,y^3-x^3\,y\right)\left(U_x\right)+2\,x\,z^2\,U^3+\left(-5\,x\,z^2+x\,y^2+x^3\right)U^2+\left(2\,x\,z^2-2\,x\,y^2-2\,x^3\right)U+x\,z^2+x\,y^2+x^3\right)\left(U_z\right)+\left(-x\,y^2\,z\,U^2+\left(x\,z^3+\left(2\,x\,y^2+x^3\right)z\right)U-x\,z^3+\left(-x\,y^2-x^3\right)z\right)\left(U_{y\,y}\right)+\left(\left(-x\,y\,z^2+x\,y^3+x^3\,y\right)U^2+\left(-2\,x\,y^3-2\,x^3\,y\right)U+x\,y\,z^2+x\,y^3+x^3\,y\right)\left(U_{y\,z}\right)+\left(x\,y^2\,z\,U-x\,z^3+\left(-x\,y^2-x^3\right)z\right)\left(U_y\right)^2+\left(\left(\left(\left(x^2\,y-y^3\right)z-y\,z^3\right)U+y\,z^3+\left(y^3+x^2\,y\right)z\right)\left(U_x\right)+2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U\right)\left(U_y\right)+\left(\left(x\,z^3+x\,y^2\,z\right)U^2+\left(-x\,z^3-x\,y^2\,z\right)U\right)\left(U_{x\,x}\right)+\left(\left(z^4+y^2\,z^2+x^2\,y^2+x^4\right)U^2+\left(-2\,z^4+\left(-2\,y^2-2\,x^2\right)z^2-2\,x^2\,y^2-2\,x^4\right)U+z^4+\left(y^2+2\,x^2\right)z^2+x^2\,y^2+x^4\right)\left(U_{x\,z}\right)+\left(\left(y\,z^3+\left(y^3-x^2\,y\right)z\right)U^2+\left(-2\,y\,z^3-2\,y^3\,z\right)U+y\,z^3+\left(y^3+x^2\,y\right)z\right)\left(U_{x\,y}\right)+\left(-x\,z^3-x\,y^2\,z\right)U\left(U_x\right)^2+\left(2\,x^2\,z\,U^3+\left(z^3+\left(y^2-5\,x^2\right)z\right)U^2+\left(\left(2\,x^2-2\,y^2\right)z-2\,z^3\right)U+z^3+\left(y^2+x^2\right)z\right)\left(U_x\right)+2\,x\,z\,U^4-6\,x\,z\,U^3+6\,x\,z\,U^2-2\,x\,z\,U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^3+\left(-6\,z^4+\left(-12\,y^2-12\,x^2\right)z^2-6\,y^4-12\,x^2\,y^2-6\,x^4\right)U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right)$$

(%i73) ratsimp(ric[3,4]);

$$(\%o73)\ -((\left((y^3+x^2\,y)\,z\,U^2+(-y^3-x^2\,y)\,z\,U\right)(U_{zz})+(-y^3-x^2\,y)\,z\,U\,(U_z)^2+((\left((2\,y^2+x^2)\,z^2+x^2\,y^2\right)(U_x)+2\,y\,z^2\,U^3+(-5\,y\,z^2+y^3+x^2\,y)\,U^2+(2\,y\,z^2-2\,y^3-2\,x^2\,y)\,U+y\,z^2+y^3+x^2\,y)\,(U_z)+\left((y\,z^3+x^2\,y\,z)\,U^2+(-y\,z^3-x^2\,y\,z)\,U\right)(U_{yy})+((z^4+x^2\,z^2+y^4+x^2\,y^2)\,U^2+\left(-2\,z^4+(-2\,y^2-2\,x^2)\,z^2-2\,y^4-2\,x^2\,y^2\right)U+z^4+(2\,y^2+x^2)\,z^2+y^4+x^2\,y^2)\,(U_{yz})+(-y\,z^3-x^2\,y\,z)\,(U_y)^2+((\left((x\,y^2-x^3)\,z-x\,z^3\right)U+x\,z^3+(x\,y^2+x^3)\,z\right)(U_x)+2\,y^2\,z\,U^3+\left(z^3+(x^2-5\,y^2)\,z\right)U^2+\left((2\,y^2-2\,x^2)\,z-2\,z^3\right)U+z^3+(y^2+x^2)\,z)\,(U_y)+\left(-x^2\,y\,z\,U^2+\left(y\,z^3+(y^3+2\,x^2\,y)\,z\right)U-y\,z^3+(-y^3-x^2\,y)\,z\right)(U_{xx})+\left((-x\,y\,z^2+x\,y^3+x^3\,y)\,U^2+(-2\,x\,y^3-2\,x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y\right)(U_{xz})+\left(\left(x\,z^3+(x^3-x\,y^2)\,z\right)U^2+(-2\,x\,z^3-2\,x^3\,z)\,U+x\,z^3+(x\,y^2+x^3)\,z\right)(U_{xy})+\left(x^2\,y\,z\,U-y\,z^3+(-y^3-x^2\,y)\,z\right)(U_x)^2+(2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U)\,(U_x)+2\,y\,z\,U^4-6\,y\,z\,U^3+6\,y\,z\,U^2-2\,y\,z\,U)\,/\,(\left(2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^3+\left(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4\right)U^2+\left(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

Ora mi preparo a specificare cosa deve valere la funzione U(x,y,z) per fare in modo che la metrica sia quella di un buco nero neutro ossia sia la metrica di Schwarzschild.

(%i74) v0n:2*m/rq^(1/2);

$$(\%o74)\ \frac{2\,m}{\sqrt{z^2+y^2+x^2}}$$

Specifico tutte le derivate di primo e secondo ordine.

(%i75) vxn:ratsimp(diff(v0n,x));

$$(\%o75)\ -\frac{2\,m\,x}{\left(z^2+y^2+x^2\right)^{3/2}}$$

(%i76) vyn:ratsimp(diff(v0n,y))$

(%i77) vzn:ratsimp(diff(v0n,z))$

(%i78) vxxn:ratsimp(diff(vxn,x));

$$(\%o78)\ -\frac{\sqrt{z^2+y^2+x^2}\,(2\,m\,z^2+2\,m\,y^2-4\,m\,x^2)}{z^6+(3\,y^2+3\,x^2)\,z^4+(3\,y^4+6\,x^2\,y^2+3\,x^4)\,z^2+y^6+3\,x^2\,y^4+3\,x^4\,y^2+x^6}$$

(%i79) vyyn:ratsimp(diff(vyn,y))$

(%i80) vzzn:ratsimp(diff(vzn,z))$

(%i81) vxyn:ratsimp(diff(vxn,y));

$$(\%o81)\ \frac{6\,m\,x\,y}{\sqrt{z^2+y^2+x^2}\,(z^4+(2\,y^2+2\,x^2)\,z^2+y^4+2\,x^2\,y^2+x^4)}$$

(%i82) vxzn:ratsimp(diff(vxn,z))$

(%i83) vyzn:ratsimp(diff(vyn,z))$

Ora sostituisco alle derivate alcuni simboli ossia uso v0, vx, vy, vz, vxx,
vxy, vzz, vxy, vxz, vyz.
Inizio la procedura con ric[1,1]

(%i84) rics:ratsimp(ric[1,1]);

(%o84) $$(\left(z^2\,U^2+\left(-2\,z^2-y^2-x^2\right)U+z^2+y^2+x^2\right)\left(U_{z\,z}\right)+\left(y^2+x^2\right)\left(U_z\right)^2+$$
$$\left(-2\,y\,z\left(U_y\right)-2\,x\,z\left(U_x\right)+2\,z\,U^2-2\,z\,U\right)\left(U_z\right)+\left(y^2\,U^2+\left(-z^2-2\,y^2-x^2\right)U+z^2+y^2+x^2\right)\left(U_{y\,y}\right)+$$
$$\left(2\,y\,z\,U^2-2\,y\,z\,U\right)\left(U_{y\,z}\right)+\left(z^2+x^2\right)\left(U_y\right)^2+\left(-2\,x\,y\left(U_x\right)+2\,y\,U^2-2\,y\,U\right)\left(U_y\right)+$$
$$\left(x^2\,U^2+\left(-z^2-y^2-2\,x^2\right)U+z^2+y^2+x^2\right)\left(U_{x\,x}\right)+\left(2\,x\,z\,U^2-2\,x\,z\,U\right)\left(U_{x\,z}\right)+\left(2\,x\,y\,U^2-2\,x\,y\,U\right)\left(U_{x\,y}\right)+$$
$$\left(z^2+y^2\right)\left(U_x\right)^2+\left(2\,x\,U^2-2\,x\,U\right)\left(U_x\right))\,/\,\left(\left(2\,z^2+2\,y^2+2\,x^2\right)U-2\,z^2-2\,y^2-2\,x^2\right)$$

(%i85) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i86) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i87) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i88) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i89) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i90) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i91) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i92) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i93) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i94) rica:ratsimp(subst(v0,U,ric9));

(%o94) $$(\left(\left(v0^2-2\,v0+1\right)vzz+\left(1-v0\right)vyy+vy^2+\left(1-v0\right)vxx+vx^2\right)z^2+$$
$$\left(\left(\left(2\,v0^2-2\,v0\right)vyz-2\,vy\,vz\right)y+\left(\left(2\,v0^2-2\,v0\right)vxz-2\,vx\,vz\right)x+\left(2\,v0^2-2\,v0\right)vz\right)z+$$
$$\left(\left(1-v0\right)vzz+vz^2+\left(v0^2-2\,v0+1\right)vyy+\left(1-v0\right)vxx+vx^2\right)y^2+$$
$$\left(\left(\left(2\,v0^2-2\,v0\right)vxy-2\,vx\,vy\right)x+\left(2\,v0^2-2\,v0\right)vy\right)y+$$
$$\left(\left(1-v0\right)vzz+vz^2+\left(1-v0\right)vyy+vy^2+\left(v0^2-2\,v0+1\right)vxx\right)x^2+\left(2\,v0^2-2\,v0\right)vx\,x)\,/\,\left(\left(2\,v0-2\right)z^2+$$
$$\left(2\,v0-2\right)y^2+\left(2\,v0-2\right)x^2\right)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i95) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i96) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i97) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i98) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i99) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i100) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i101) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i102) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i103) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i104) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?

(%i105) ratsimp(rican);

(%o105) 0

Ripeto tutta la procedura con ric[2,2]

(%i106) rics:ratsimp(ric[2,2]);

(%o106)
$$\left(x^2\,z^2\,U^2+\left(-2\,x^2\,z^2-x^2\,y^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_{zz}\right)+$$
$$\left(-x^2\,z^2\,U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_z\right)^2+\left(-2\,x^2\,y\,z\,U\left(U_y\right)+\right.$$
$$\left.\left(\left(2\,x\,z^3+2\,x\,y^2\,z\right)U-2\,x\,z^3+\left(-2\,x\,y^2-2\,x^3\right)z\right)\left(U_x\right)+\left(2\,z^3+2\,y^2\,z\right)U^3+\left(-6\,z^3-6\,y^2\,z\right)U^2+\right.$$
$$\left.\left(6\,z^3+\left(6\,y^2+2\,x^2\right)z\right)U-2\,z^3+\left(-2\,y^2-2\,x^2\right)z\right)\left(U_z\right)+$$
$$\left(x^2\,y^2\,U^2+\left(-x^2\,z^2-2\,x^2\,y^2-x^4\right)U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_{yy}\right)+\left(2\,x^2\,y\,z\,U^2-2\,x^2\,y\,z\,U\right)\left(U_{yz}\right)+$$
$$\left(-x^2\,y^2\,U+x^2\,z^2+x^2\,y^2+x^4\right)\left(U_y\right)^2+\left(\left(\left(2\,x\,y\,z^2+2\,x\,y^3\right)U-2\,x\,y\,z^2-2\,x\,y^3-2\,x^3\,y\right)\left(U_x\right)+\right.$$
$$\left.\left(2\,y\,z^2+2\,y^3\right)U^3+\left(-6\,y\,z^2-6\,y^3\right)U^2+\left(6\,y\,z^2+6\,y^3+2\,x^2\,y\right)U-2\,y\,z^2-2\,y^3-2\,x^2\,y\right)\left(U_y\right)+$$
$$\left(\left(-2\,x^2\,z^2-2\,x^2\,y^2-x^4\right)U^2+\left(3\,x^2\,z^2+3\,x^2\,y^2+2\,x^4\right)U-x^2\,z^2-x^2\,y^2-x^4\right)\left(U_{xx}\right)+$$
$$\left(\left(-2\,x\,z^3-2\,x\,y^2\,z\right)U^2+\left(4\,x\,z^3+\left(4\,x\,y^2+2\,x^3\right)z\right)U-2\,x\,z^3+\left(-2\,x\,y^2-2\,x^3\right)z\right)\left(U_{xz}\right)+$$
$$\left(\left(-2\,x\,y\,z^2-2\,x\,y^3\right)U^2+\left(4\,x\,y\,z^2+4\,x\,y^3+2\,x^3\,y\right)U-2\,x\,y\,z^2-2\,x\,y^3-2\,x^3\,y\right)\left(U_{xy}\right)+$$
$$\left(\left(-z^4-2\,y^2\,z^2-y^4\right)U+z^4+\left(2\,y^2+x^2\right)z^2+y^4+x^2\,y^2\right)\left(U_x\right)^2+$$
$$\left(\left(2\,x\,z^2+2\,x\,y^2\right)U^3+\left(-8\,x\,z^2-8\,x\,y^2-2\,x^3\right)U^2+\left(10\,x\,z^2+10\,x\,y^2+6\,x^3\right)U-4\,x\,z^2-4\,x\,y^2-4\,x^3\right)$$
$$\left(U_x\right)+\left(2\,z^2+2\,y^2\right)U^4+\left(-6\,z^2-6\,y^2\right)U^3+\left(6\,z^2+6\,y^2\right)U^2+\left(-2\,z^2-2\,y^2\right)U\Big)\,/\,\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2\right.\right.$$
$$\left.\left.U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right.\right.$$

(%i107) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i108) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i109) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i110) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i111) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i112) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i113) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i114) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i115) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i116) rica:ratsimp(subst(v0,U,ric9));

(%o116)
$$-((v0-1)\,vx^2\,z^4+\left(\left((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz\right)x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz\right)z^3+((2\,v0-2)\,vx^2\,y^2+\left(\left((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy\right)x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy\right)y+\left((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(v0-1)\,vyy-vy^2+(2\,v0^2-3\,v0+1)\,vxx-vx^2\right)x^2+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,z^2+(\left(\left((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+$$
$$\left(2\,v0\,vy\,vz+(2\,v0-2\,v0^2)\,vyz\right)x^2\,y+(2\,vx\,vz+(2-2\,v0)\,vxz)\,x^3+(2-2\,v0)\,vz\,x^2)\,z+(v0-1)\,vx^2\,y^4+\left(\left((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy\right)x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy\right)y^3+(\left((v0-1)\,vzz-v$$
$$(-2\,v0^3+8\,v0^2-10\,v0+4)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,y^2+\left((2\,vx\,vy+(2-2\,v0)\,vxy)\,x^3+(2-2\,v0)\,vy\,x^2\right)y+\left((v0-1)\,vzz-vz^2+(v0-1)\,vyy-vy^2+(v0^2-2\,v0+1)\,vxx\right)x^4+(2\,v0^2-6\,v0+4)\,vx\,x^3)/((2\,v0^3-6$$
$$(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i117) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i118) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i119) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i120) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i121) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i122) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i123) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i124) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i125) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i126) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?

(%i127) ratsimp(rican);

(%o127) 0

Ripeto tutta la procedura con ric[3,3]

(%i128) rics:ratsimp(ric[3,3]);

(%o128) $$\left(y^2\,z^2\,U^2+\left(-2\,y^2\,z^2-y^4-x^2\,y^2\right)U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_{z\,z}\right)+$$
$$\left(-y^2\,z^2\,U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_z\right)^2+\left(\left(2\,y\,z^3+2\,x^2\,y\,z\right)U-2\,y\,z^3+\left(-2\,y^3-2\,x^2\,y\right)z\right)\left(U_y\right)-2\,x\,y^2$$
$$z\,U\left(U_x\right)+\left(2\,z^3+2\,x^2\,z\right)U^3+\left(-6\,z^3-6\,x^2\,z\right)U^2+\left(6\,z^3+\left(2\,y^2+6\,x^2\right)z\right)U-2\,z^3+\left(-2\,y^2-2\,x^2\right)z\,)$$
$$\left(U_z\right)+\left(\left(-2\,y^2\,z^2-y^4-2\,x^2\,y^2\right)U^2+\left(3\,y^2\,z^2+2\,y^4+3\,x^2\,y^2\right)U-y^2\,z^2-y^4-x^2\,y^2\right)\left(U_{y\,y}\right)+$$
$$\left(\left(-2\,y\,z^3-2\,x^2\,y\,z\right)U^2+\left(4\,y\,z^3+\left(2\,y^3+4\,x^2\,y\right)z\right)U-2\,y\,z^3+\left(-2\,y^3-2\,x^2\,y\right)z\right)\left(U_{y\,z}\right)+$$
$$\left(\left(-z^4-2\,x^2\,z^2-x^4\right)U+z^4+\left(y^2+2\,x^2\right)z^2+x^2\,y^2+x^4\right)\left(U_y\right)^2+\left(\left(2\,x\,y\,z^2+2\,x^3\,y\right)U-2\,x\,y\,z^2-2\,x\,y^3\right.$$
$$U^2+\left(10\,y\,z^2+6\,y^3+10\,x^2\,y\right)U-4\,y\,z^2-4\,y^3-4\,x^2\,y\,)\left(U_y\right)+$$
$$\left(x^2\,y^2\,U^2+\left(-y^2\,z^2-y^4-2\,x^2\,y^2\right)U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_{x\,x}\right)+\left(2\,x\,y^2\,z\,U^2-2\,x\,y^2\,z\,U\right)\left(U_{x\,z}\right)+$$
$$\left(\left(-2\,x\,y\,z^2-2\,x^3\,y\right)U^2+\left(4\,x\,y\,z^2+2\,x\,y^3+4\,x^3\,y\right)U-2\,x\,y\,z^2-2\,x\,y^3-2\,x^3\,y\right)\left(U_{x\,y}\right)+$$
$$\left(-x^2\,y^2\,U+y^2\,z^2+y^4+x^2\,y^2\right)\left(U_x\right)^2+$$
$$\left(\left(2\,x\,z^2+2\,x^3\right)U^3+\left(-6\,x\,z^2-6\,x^3\right)U^2+\left(6\,x\,z^2+2\,x\,y^2+6\,x^3\right)U-2\,x\,z^2-2\,x\,y^2-2\,x^3\right)\left(U_x\right)+$$
$$\left(2\,z^2+2\,x^2\right)U^4+\left(-6\,z^2-6\,x^2\right)U^3+\left(6\,z^2+6\,x^2\right)U^2+\left(-2\,z^2-2\,x^2\right)U\,)\,/\,\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4\right.\right.$$
$$U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4$$

(%i129) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i130) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i131) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i132) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i133) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i134) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i135) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i136) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i137) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i138) rica:ratsimp(subst(v0,U,ric9));

(%o138) $$-((v0-1)\,vy^2\,z^4+\left(\left((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz\right)y+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz\right)z^3+(\left((-v0^2+2\,v0-1)\,vzz\right.$$
$$\left.\left(\left((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy\right)x+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vy\right)y+(2\,v0-2)\,vy^2\,x^2+\right.$$
$$\left.(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0\right)z^2+((2\,vy\,vz+(2-2\,v0)\,vyz)\,y^3+$$
$$\left(\left(2\,v0\,vx\,vz+(2\,v0-2\,v0^2)\,vxz\right)x+(2-2\,v0)\,vz\right)y^2+\left((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz\right)x^2\,y+$$
$$(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz\,x^2)\,z+\left((v0-1)\,vzz-vz^2+(v0^2-2\,v0+1)\,vyy+(v0-1)\,vxx-vx^2\right)y^4+$$
$$\left((2\,vx\,vy+(2-2\,v0)\,vxy)\,x+(2\,v0^2-6\,v0+4)\,vy\right)y^3+(\left((v0-1)\,vzz-vz^2+(2\,v0^2-3\,v0+1)\,vyy-vy\right.$$
$$\left.x^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x^3+(-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,x^2\right)/((2\,v0^3-6\,v0^2+6\,v0-2)$$
$$z^4+\left((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\right)z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+$$
$$(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i139) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i140) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i141) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i142) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i143) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i144) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i145) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i146) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i147) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i148) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?

(%i149) ratsimp(rican);

(%o149) 0

Ripeto tutta la procedura con ric[4,4]

(%i150) rics:ratsimp(ric[4,4]);

(%o150) $$-(\left(\left(z^4+\left(2\,y^2+2\,x^2\right)z^2\right)U^2+\left(\left(-3\,y^2-3\,x^2\right)z^2-2\,z^4\right)U+z^4+\left(y^2+x^2\right)z^2\right)\left(U_{zz}\right)+$$
$$\left(\left(y^4+2\,x^2\,y^2+x^4\right)U+\left(-y^2-x^2\right)z^2-y^4-2\,x^2\,y^2-x^4\right)\left(U_z\right)^2+(\left(\left(-2\,y^3-2\,x^2\,y\right)z\,U+2\,y\,z^3+\left(2\,y^3+2\right.$$
$$\left.\left(U_x\right)+\left(-2\,y^2-2\,x^2\right)z\,U^3+\left(2\,z^3+\left(8\,y^2+8\,x^2\right)z\right)U^2+\left(\left(-10\,y^2-10\,x^2\right)z-6\,z^3\right)U+4\,z^3+\left(4\,y^2+4\,x^2\right)\right.$$
$$\left.z\right)\left(U_z\right)+\left(-y^2\,z^2\,U^2+\left(z^4+\left(2\,y^2+x^2\right)z^2\right)U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_{yy}\right)+$$
$$\left(\left(2\,y^3+2\,x^2\,y\right)z\,U^2+\left(\left(-4\,y^3-4\,x^2\,y\right)z-2\,y\,z^3\right)U+2\,y\,z^3+\left(2\,y^3+2\,x^2\,y\right)z\right)\left(U_{yz}\right)+$$
$$\left(y^2\,z^2\,U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_y\right)^2+(2\,x\,y\,z^2\,U\left(U_x\right)+\left(-2\,y^3-2\,x^2\,y\right)U^3+\left(6\,y^3+6\,x^2\,y\right)U^2+$$
$$\left(-2\,y\,z^2-6\,y^3-6\,x^2\,y\right)U+2\,y\,z^2+2\,y^3+2\,x^2\,y\right)\left(U_y\right)+$$
$$\left(-x^2\,z^2\,U^2+\left(z^4+\left(y^2+2\,x^2\right)z^2\right)U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_{xx}\right)+$$
$$\left(\left(2\,x\,y^2+2\,x^3\right)z\,U^2+\left(\left(-4\,x\,y^2-4\,x^3\right)z-2\,x\,z^3\right)U+2\,x\,z^3+\left(2\,x\,y^2+2\,x^3\right)z\right)\left(U_{xz}\right)+$$
$$\left(2\,x\,y\,z^2\,U-2\,x\,y\,z^2\,U^2\right)\left(U_{xy}\right)+\left(x^2\,z^2\,U-z^4+\left(-y^2-x^2\right)z^2\right)\left(U_x\right)^2+$$
$$\left(\left(-2\,x\,y^2-2\,x^3\right)U^3+\left(6\,x\,y^2+6\,x^3\right)U^2+\left(-2\,x\,z^2-6\,x\,y^2-6\,x^3\right)U+2\,x\,z^2+2\,x\,y^2+2\,x^3\right)\left(U_x\right)+$$
$$\left(-2\,y^2-2\,x^2\right)U^4+\left(6\,y^2+6\,x^2\right)U^3+\left(-6\,y^2-6\,x^2\right)U^2+\left(2\,y^2+2\,x^2\right)U)\,/\,(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4\right.$$
$$\left.U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right.$$

(%i151) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i152) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i153) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i154) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i155) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i156) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i157) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i158) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i159) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i160) rica:ratsimp(subst(v0,U,ric9));

(%o160) $$-\left(\left(v0^2-2\,v0+1\right)vzz+\left(v0-1\right)vyy-vy^2+\left(v0-1\right)vxx-vx^2\right)z^4+\left(\left(2\,vy\,vz+\left(2-2\,v0\right)vyz\right)y+\left(2\,vx\,vz+\left(2-2\,v0\right)vxz\right)x+\left(2\,v0^2-6\,v0+4\right)vz\right)z^3+\left(\left(\left(2\,v0\,vx\,vy+\left(2\,v0-2\,v0^2\right)vxy\right)x+\left(2-2\,v0\right)vy\right)y+\left(\left(2\,v0^2-3\,v0+1\right)vzz-vz^2+\left(v0-1\right)vyy-vy^2+\left(-v0^2+2\,v0-1\right)vxx+\left(v0-1\right)vx^2\right)x^2+\left(2-2\,v0\right)vx\,z\right)$$

$$\left(\left(\left(2-2\,v0\right)vx\,vz+\left(2\,v0^2-4\,v0+2\right)vxz\right)x+\left(-2\,v0^3+8\,v0^2-10\,v0+4\right)vz\right)y^2+\left(\left(2-2\,v0\right)vy\,vz+\left(2\,v0^2-4\,v0+2\right)vyz\right)x^2\,y+\left(\left(2-2\,v0\right)vx\,vz+\left(2\,v0^2-4\,v0+2\right)vxz\right)x^3+\left(-2\,v0^3+8\,v0^2-10\,v0+4\right)vz\,x^2\,z+\left(v0-1\right)vz^2\,y^4+\left(-2\,v0^3+6\,v0^2-6\,v0+2\right)vy\,y^3+\left(\left(2\,v0-2\right)vz^2\,x^2+\left(-2\,v0^3+6\,v0^2-6\,v0+2\right)vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0\right)y^2+\left(-2\,v0^3+6\,v0^2-6\,v0+2\right)vy\,x^2\,y+\left(v0-1\right)vz^2\,x^4+\left(-2\,v0^3+6\,v0^2-6\,v0+2\right)vx\,x^3+\left(-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0\right)x^2\Big)\Big/\left(\left(2\,v0^3-6\,v0^2+6\,v0-2\right)z^4+\left(\left(4\,v0^3-12\,v0^2+12\,v0-4\right)y^2+\left(4\,v0^3-12\,v0^2+12\,v0-4\right)x^2\right)z^2+\left(2\,v0^3-6\,v0^2+6\,v0-2\right)y^4+\left(4\,v0^3-12\,v0^2+12\,v0-4\right)x^2\,y^2+\left(2\,v0^3-6\,v0^2+6\,v0-2\right)x^4\right)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i161) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i162) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i163) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i164) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i165) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i166) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i167) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i168) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i169) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i170) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?

(%i171) ratsimp(rican);

(%o171) 0

Ora verifico anche i termini NON DIAGONALI che possono essere diversi
da zero se la funzione non possiede le opportune caratteristiche.

(%i172) rics:ratsimp(ric[2,3]);

(%o172) $$(x\,y\,z^2\,U^2+(-2\,x\,y\,z^2-x\,y^3-x^3\,y)\,U+x\,y\,z^2+x\,y^3+x^3\,y)\,(U_{z\,z})+$$
$$(-x\,y\,z^2\,U+x\,y\,z^2+x\,y^3+x^3\,y)\,(U_z)^2+((x\,z^3+(x^3-x\,y^2)\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)\,(U_y)+$$
$$((y\,z^3+(y^3-x^2\,y)\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)\,(U_x)-2\,x\,y\,z\,U^3+6\,x\,y\,z\,U^2-4\,x\,y\,z\,U)\,(U_z)+$$
$$((-x\,y\,z^2-x^3\,y)\,U^2+(x\,y\,z^2+x^3\,y)\,U)\,(U_{y\,y})+$$
$$(((x\,y^2-x^3)\,z-x\,z^3)\,U^2+(2\,x\,z^3+2\,x^3\,z)\,U-x\,z^3+(-x\,y^2-x^3)\,z)\,(U_{y\,z})+(x\,y\,z^2+x^3\,y)\,U\,(U_y)^2+(((-$$
$$(2\,x\,z^2-2\,x\,y^2+2\,x^3)\,U-x\,z^2-x\,y^2-x^3)\,(U_y)+((-x\,y\,z^2-x\,y^3)\,U^2+(x\,y\,z^2+x\,y^3)\,U)\,(U_{x\,x})+$$
$$(((x^2\,y-y^3)\,z-y\,z^3)\,U^2+(2\,y\,z^3+2\,y^3\,z)\,U-y\,z^3+(-y^3-x^2\,y)\,z)\,(U_{x\,z})+(((-y^2-x^2)\,z^2-y^4-x^4)$$
$$U^2+((2\,y^2+2\,x^2)\,z^2+2\,y^4+2\,x^2\,y^2+2\,x^4)\,U+(-y^2-x^2)\,z^2-y^4-2\,x^2\,y^2-x^4)\,(U_{x\,y})+(x\,y\,z^2+x\,y^3)$$
$$U\,(U_x)^2+(-2\,x^2\,y\,U^3+(-y\,z^2-y^3+5\,x^2\,y)\,U^2+(2\,y\,z^2+2\,y^3-2\,x^2\,y)\,U-y\,z^2-y^3-x^2\,y)\,(U_x)-2\,x\,y$$
$$U^4+6\,x\,y\,U^3-6\,x\,y\,U^2+2\,x\,y\,U)\,/\,((2\,z^4+(4\,y^2+4\,x^2)\,z^2+2\,y^4+4\,x^2\,y^2+2\,x^4)\,U^3+$$
$$(-6\,z^4+(-12\,y^2-12\,x^2)\,z^2-6\,y^4-12\,x^2\,y^2-6\,x^4)\,U^2+$$
$$(6\,z^4+(12\,y^2+12\,x^2)\,z^2+6\,y^4+12\,x^2\,y^2+6\,x^4)\,U-2\,z^4+(-4\,y^2-4\,x^2)\,z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

(%i173) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i174) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i175) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i176) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i177) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i178) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i179) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i180) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i181) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i182) rica:ratsimp(subst(v0,U,ric9));

(%o182) $$-((v0-1)\,vx\,vy\,z^4+\left(\left((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz\right)y+\left((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz\right)x\right)z^3+(\left((v0-1)\,vx\,vy+(v0^2-2\,v0+1)\,vx\right)y+\left((v0-1)\,vx\,vy+(v0^2-2\,v0+1)\,vxy\right)x^2+(v0^2-2\,v0+1)\,vy\,x)\,z^2+(\left((1-v0)\,v\right.$$

$$\left.\left((v0+1)\,vx\,vz+(1-v0^2)\,vxz\right)x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vz\,x\right)y+\left((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz\right.$$

$$\left.x^3\right)z+(v0^2-2\,v0+1)\,vxy\,y^4+\left(\left((v0-1)\,vzz-vz^2+(v0^2-v0)\,vxx-v0\,vx^2\right)x+(v0^2-2\,v0+1)\,vx\right)y^3$$

$$+\left((2\,v0\,vx\,vy+(2-2\,v0)\,vxy\right)x^2+(2\,v0^3-5\,v0^2+2\,v0+1)\,vy\,x\Big)y^2+(\left((v0-1)\,vzz-vz^2+(v0^2-v0)\right.$$

$$\left.(2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,x\right)y+(v0^2-2\,v0+1)\,vxy\,x^4+(v0^2-2\,v0+1)\,vy\,x^3)/((2\,v0^3-6\,v0^2+$$

$$(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i183) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i184) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i185) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i186) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i187) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i188) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i189) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i190) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i191) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i192) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?

(%i193) ratsimp(rican);

(%o193) 0

Verifico anche il termine NON DIAGONALE r[2,4] che puo' essere diverso
da zero se la funzione non possiede le opportune caratteristiche.

(%i194) rics:ratsimp(ric[2,4]);

(%o194) $$-\left(\left(\left(x\,y^2+x^3\right)z\,U^2+\left(-x\,y^2-x^3\right)z\,U\right)\left(U_{z\,z}\right)+\left(-x\,y^2-x^3\right)z\,U\left(U_z\right)^2+\left(\left(\left(x\,y\,z^2-x\,y^3-x^3\,y\right)\left(U_x\right)+2\,x\,z^2\,U^3+\left(-5\,x\,z^2+x\,y^2+x^3\right)U^2+\left(2\,x\,z^2-2\,x\,y^2-2\,x^3\right)U+x\,z^2+x\,y^2+x^3\right)\left(U_z\right)+\left(-x\,y^2\,z\,U^2+\left(x\,z^3+\left(2\,x\,y^2+x^3\right)z\right)U-x\,z^3+\left(-x\,y^2-x^3\right)z\right)\left(U_{y\,y}\right)+\left(\left(-x\,y\,z^2+x\,y^3+x^3\,y\right)U^2+\left(-2\,x\,y^3-2\,x^3\,y\right)U+x\,y\,z^2+x\,y^3+x^3\,y\right)\left(U_{y\,z}\right)+\left(x\,y^2\,z\,U-x\,z^3+\left(-x\,y^2-x^3\right)z\right)\left(U_y\right)^2+\left(\left(\left(\left(x^2\,y-y^3\right)z-y\,z^3\right)U+y\,z^3+\left(y^3+x^2\,y\right)z\right)\left(U_x\right)+2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U\right)\left(U_y\right)+\left(\left(x\,z^3+x\,y^2\,z\right)U^2+\left(-x\,z^3-x\,y^2\,z\right)U\right)\left(U_{x\,x}\right)+\left(\left(z^4+y^2\,z^2+x^2\,y^2+x^4\right)U^2+\left(-2\,z^4+\left(-2\,y^2-2\,x^2\right)z^2-2\,x^2\,y^2-2\,x^4\right)U+z^4+\left(y^2+2\,x^2\right)z^2+x^2\,y^2+x^4\right)\left(U_{x\,z}\right)+\left(\left(\left(y\,z^3+\left(y^3-x^2\,y\right)z\right)U^2+\left(-2\,y\,z^3-2\,y^3\,z\right)U+y\,z^3+\left(y^3+x^2\,y\right)z\right)\left(U_{x\,y}\right)+\left(-x\,z^3-x\,y^2\,z\right)U\left(U_x\right)^2+\left(2\,x^2\,z\,U^3+\left(z^3+\left(y^2-5\,x^2\right)z\right)U^2+\left(\left(2\,x^2-2\,y^2\right)z-2\,z^3\right)U+z^3+\left(y^2+x^2\right)z\right)\left(U_x\right)+2\,x\,z\,U^4-6\,x\,z\,U^3+6\,x\,z\,U^2-2\,x\,z\,U\right)/\left(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^3+\left(-6\,z^4+\left(-12\,y^2-12\,x^2\right)z^2-6\,y^4-12\,x^2\,y^2-6\,x^4\right)U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4\right)$$

(%i195) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i196) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i197) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i198) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i199) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i200) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i201) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i202) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i203) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i204) rica:ratsimp(subst(v0,U,ric9));

(%o204) $$-((v0^2-2\,v0+1)\,vxz\,z^4+(((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y+(((v0-1)\,vyy-vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,z^3+(((v0-1)\,vx\,vz+(v0^2-2\,v0)\,vxz)\,x^2+(2\,v0^3-5\,v0^2+2\,v0+1)\,vz\,x)\,z^2+(((1-v0)\,vx\,vy+(v0^2-2\,v0)\,vxy)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vy\,x)\,y+(((v0^2-v0)\,vzz-v0\,vz^2+(v0-1)\,vyy-vy^2)\,x^3+(2\,v0^3-5\,v0^2+2\,v0+1)\,vx\,x^2+(2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,x)\,z+(v0-1)\,vx\,vz\,y^4+(((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x\,y^3+((((v0-1)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x^2+(v0^2-2\,v0+1)\,vz\,x)\,y^2+(((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^3\,y+(v0^2-2\,v0+1)\,vxz\,x^4+(v0^2-2\,v0+1)\,vz\,x^3)/((2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i205) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i206) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i207) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i208) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i209) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i210) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i211) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i212) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i213) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i214) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?

(%i215) ratsimp(rican);

(%o215) 0

Ora verifico anche il termine NON DIAGONALE r[3,4] che potrebbe essere diverso
da zero se la funzione non possiede le opportune caratteristiche.

(%i216) rics:ratsimp(ric[3,4]);

(%o216) $$-((\left(\left(y^3+x^2\,y\right)z\,U^2+\left(-y^3-x^2\,y\right)z\,U\right)\left(U_{z\,z}\right)+\left(-y^3-x^2\,y\right)z\,U\left(U_z\right)^2+((\left(\left(2\,y^2+x^2\right)z^2+x^2\,y^2\right)\left(U_x\right)+2\,y\,z^2\,U^3+\left(-5\,y\,z^2+y^3+x^2\,y\right)U^2+\left(2\,y\,z^2-2\,y^3-2\,x^2\,y\right)U+y\,z^2+y^3+x^2\,y\right)\left(U_z\right)+\left(\left(y\,z^3+x^2\,y\,z\right)U^2+\left(-y\,z^3-x^2\,y\,z\right)U\right)\left(U_{y\,y}\right)+((z^4+x^2\,z^2+y^4+x^2\,y^2)\,U^2+\left(-2\,z^4+\left(-2\,y^2-2\,x^2\right)z^2-2\,y^4-2\,x^2\,y^2\right)U+z^4+\left(2\,y^2+x^2\right)z^2+y^4+x^2\,y^2)\left(U_{y\,z}\right)+\left(-y\,z^3-x^2\,y\,z\right)\left(U_y\right)^2+((\left(\left(x\,y^2-x^3\right)z-x\,z^3\right)U+x\,z^3+\left(x\,y^2+x^3\right)z\right)\left(U_x\right)+2\,y^2\,z\,U^3+\left(z^3+\left(x^2-5\,y^2\right)z\right)U^2+\left(\left(2\,y^2-2\,x^2\right)z-2\,z^3\right)U+z^3+\left(y^2+x^2\right)z)\left(U_y\right)+\left(-x^2\,y\,z\,U^2+\left(y\,z^3+\left(y^3+2\,x^2\,y\right)z\right)U-y\,z^3+\left(-y^3-x^2\,y\right)z\right)\left(U_{x\,x}\right)+\left(\left(-x\,y\,z^2+x\,y^3+x^3\,y\right)U^2+\left(-2\,x\,y^3-2\,x^3\,y\right)U+x\,y\,z^2+x\,y^3+x^3\,y\right)\left(U_{x\,z}\right)+\left(\left(x\,z^3+\left(x^3-x\,y^2\right)z\right)U^2+\left(-2\,x\,z^3-2\,x^3\,z\right)U+x\,z^3+\left(x\,y^2+x^3\right)z\right)\left(U_{x\,y}\right)+\left(x^2\,y\,z\,U-y\,z^3+\left(-y^3-x^2\,y\right)z\right)\left(U_x\right)^2+\left(2\,x\,y\,z\,U^3-6\,x\,y\,z\,U^2+4\,x\,y\,z\,U\right)\left(U_x\right)+2\,y\,z\,U^4-6\,y\,z\,U^3+6\,y\,z\,U^2-2\,y\,z\,U)/(\left(2\,z^4+\left(4\,y^2+4\,x^2\right)z^2+2\,y^4+4\,x^2\,y^2+2\,x^4\right)U^3+\left(-6\,z^4+\left(-12\,y^2-12\,x^2\right)z^2-6\,y^4-12\,x^2\,y^2-6\,x^4\right)U^2+\left(6\,z^4+\left(12\,y^2+12\,x^2\right)z^2+6\,y^4+12\,x^2\,y^2+6\,x^4\right)U-2\,z^4+\left(-4\,y^2-4\,x^2\right)z^2-2\,y^4-4\,x^2\,y^2-2\,x^4)$$

(%i217) ric1:ratsimp(subst(vx,diff(U,x),rics))$

(%i218) ric2:ratsimp(subst(vy,diff(U,y),ric1))$

(%i219) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i220) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i221) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4))$

(%i222) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5))$

(%i223) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6))$

(%i224) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7))$

(%i225) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8))$

(%i226) rica:ratsimp(subst(v0,U,ric9));

(%o226) $$-((v0^2-2\,v0+1)\,vyz\,z^4+(((v0^2-v0)\,vyy-v0\,vy^2+(v0-1)\,vxx-vx^2)\,y+((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x+(v0^2-2\,v0+1)\,vy)\,z^3+((2\,v0\,vy\,vz+(2-2\,v0)\,vyz)\,y^2+(((v0+1)\,vx\,vz+(1-v0^2)\,vxz)\,x+(2\,v0^3-5\,v0^2+2\,v0+1)\,vz)\,y+((v0-1)\,vy\,vz+(v0^2-2\,v0+1)\,vyz\,x^2)\,z^2+(((v0^2-v0)\,vzz-v0\,vz^2+(v0-1)\,vxx-vx^2)\,y^3+(((v0+1)\,vx\,vy+(1-v0^2)\,vxy)\,x+(2\,v0^3-5\,v0^2+2\,v0+1)\,vy)\,y^2+(((v0^2-v0)\,vzz-v0\,vz^2+(v0^2-v$$

$$(2\,v0^3-6\,v0^2+4\,v0)\,vx\,x+2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,y+((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x^3+(v0^2-2\,v0+1)\,vy\,x^2)\,z+(v0^2-2\,v0+1)\,vyz\,y^4+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x+(v0^2-2\,v0+1)\,vz)\,y^3+((v0-1)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^2\,y^2+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x^3+(v0^2-2\,v0+1)\,vz\,x^2)\,y+(v0-1)\,vy\,vz\,x^4)/((2\,v0$$

$$(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i227) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i228) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i229) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i230) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i231) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i232) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i233) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i234) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i235) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i236) rican:ratsimp(subst(v0n,v0,ric9n))$

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

(%i237) ratsimp(rican);

(%o237) 0

Per finire ristampo il tensore metrico covariante che ho usato per trovare la metrica del buco nero neutro in coordinate cartesiane.

(%i238) lg;

(%o238)
$$\begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & -\frac{(z^2+y^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \frac{x\,y\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\frac{(z^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \\ 0 & \frac{x\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & \frac{y\,z\,U}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} & -\frac{(y^2+x^2)\,U-z^2-y^2-x^2}{(z^2+y^2+x^2)\,U-z^2-y^2-x^2} \end{bmatrix}$$

E per riepilogo ecco anche il tensore metrico in forma controvariante:

(%i239) ug;

(%o239)
$$\begin{bmatrix} -\frac{1}{U-1} & 0 & 0 & 0 \\ 0 & \frac{x^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \frac{x\,y\,U}{z^2+y^2+x^2} & \frac{x\,z\,U}{z^2+y^2+x^2} \\ 0 & \frac{x\,y\,U}{z^2+y^2+x^2} & \frac{y^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} & \frac{y\,z\,U}{z^2+y^2+x^2} \\ 0 & \frac{x\,z\,U}{z^2+y^2+x^2} & \frac{y\,z\,U}{z^2+y^2+x^2} & \frac{z^2\,U-z^2-y^2-x^2}{z^2+y^2+x^2} \end{bmatrix}$$

Una caratteristica fondamentale di questa metrica è il fatto che il determinante vale -1 ossia è identico a quello della metrica pseudoeuclidea.

(%i240) ratsimp(determinant(lg));

(%o240) −1

(%i241) ratsimp(determinant(ug));

(%o241) −1

Created with wxMaxima.