Prove di calcolo tensoriale

Provo a realizzare un documento wxm per il calcolo di tutte
le grandezze derivate da un assegnato tensore metrico

```
(%i1) derivabbrev:true;
(%o1) true
```

```
(%i2) depends(U,[x,y,z]);
(%o2) [U(x,y,z)]
```

```
(%i3) ax0:diff(1/U,x);
(%o3) -U_x/U^2
```

```
(%i4) ay0:diff(1/U,y);
(%o4) -U_y/U^2
```

```
(%i5) az0:diff(1/U,z);
(%o5) -U_z/U^2
```

```
(%i6) an:(ratsimp(ax0^2+ay0^2+az0^2))^(1/2);
(%o6) sqrt((U_z)^2+(U_y)^2+(U_x)^2)/U^2
```

```
(%i7) (ax1:ax0/an,ay1:ay0/an,az1:az0/an);
(%o7) -U_z/sqrt((U_z)^2+(U_y)^2+(U_x)^2)
```

```
(%i8) depends(ax,[x,y,z]);
(%o8) [ax(x,y,z)]
```

```
(%i9) depends(ay,[x,y,z]);
(%o9) [ay(x,y,z)]
```

```
(%i10) depends(az,[x,u,z]);
(%o10) [az(x,u,z)]
```

```
(%i11) ax*az;
(%o11) ax az
```

```
(%i12) ay*ay;
(%o12) ay^2
```

```
(%i13) g_ik:matrix(
       [1-U,0,0,0],
       [0,ax*ax*U/(U-1)-1,ax*ay*U/(U-1),ax*az*U/(U-1)],
       [0,ax*ay*U/(U-1),ay*ay*U/(U-1)-1,ay*az*U/(U-1)],
       [0,ax*az*U/(U-1),ay*az*U/(U-1),az*az*U/(U-1)-1]
       );
```

$$(\%o13)\quad \begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & \frac{ax^2\,U}{U-1}-1 & \frac{ax\,ay\,U}{U-1} & \frac{ax\,az\,U}{U-1} \\ 0 & \frac{ax\,ay\,U}{U-1} & \frac{ay^2\,U}{U-1}-1 & \frac{ay\,az\,U}{U-1} \\ 0 & \frac{ax\,az\,U}{U-1} & \frac{ay\,az\,U}{U-1} & \frac{az^2\,U}{U-1}-1 \end{bmatrix}$$

```
(%i14) g_IK:matrix(
       [1/(1-U),0,0,0],
       [0,ax*ax*U-1,ax*ay*U,ax*az*U],
       [0,ax*ay*U,ay*ay*U-1,ay*az*U],
       [0,ax*az*U,ay*az*U,az*az*U-1]
       );
```

$$(\%o14)\quad \begin{bmatrix} \frac{1}{1-U} & 0 & 0 & 0 \\ 0 & ax^2\,U-1 & ax\,ay\,U & ax\,az\,U \\ 0 & ax\,ay\,U & ay^2\,U-1 & ay\,az\,U \\ 0 & ax\,az\,U & ay\,az\,U & az^2\,U-1 \end{bmatrix}$$

```
(%i15) ratsimp( g_ik . g_IK);
```

(%o15)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{(ax^2\,az^2+ax^2\,ay^2+ax^4-ax^2)\,U^2+U-1}{U-1} & \frac{(ax\,ay\,az^2+ax\,ay^3+(ax^3-ax)\,ay)\,U^2}{U-1} & \frac{(ax\,az^3+(ax\,ay^2+ax^3-ax)\,az)} \\ 0 & \frac{(ax\,ay\,az^2+ax\,ay^3+(ax^3-ax)\,ay)\,U^2}{U-1} & \frac{(ay^2\,az^2+ay^4+(ax^2-1)\,ay^2)\,U^2+U-1}{U-1} & \frac{(ay\,az^3+(ay^3+(ax^2-1)\,ay)\,az)}{U-1} \\ 0 & \frac{(ax\,az^3+(ax\,ay^2+ax^3-ax)\,az)\,U^2}{U-1} & \frac{(ay\,az^3+(ay^3+(ax^2-1)\,ay)\,az)\,U^2}{U-1} & \frac{(az^4+(ay^2+ax^2-1)\,az^2)\,U^2+U}{U-1} \end{bmatrix}$$

(%i16) `ginv:ratsimp(invert(g_ik));`

(%o16)
$$\begin{bmatrix} -\frac{1}{U-1} & 0 & 0 & 0 \\ 0 & -\frac{(az^2+ay^2-1)\,U+1}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ax\,ay\,U}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ax\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} \\ 0 & \frac{ax\,ay\,U}{(az^2+ay^2+ax^2-1)\,U+1} & -\frac{(az^2+ax^2-1)\,U+1}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ay\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} \\ 0 & \frac{ax\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ay\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} & -\frac{(ay^2+ax^2-1)\,U+1}{(az^2+ay^2+ax^2-1)\,U+1} \end{bmatrix}$$

(%i17) `ratsimp(ginv .g_ik);`

(%o17)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Carico la libreria

(%i18) `(if atom(lg) then load(ctensor));`

(%o18)
C:/Programmi/Maxima-5.20.1/share/maxima/5.20.1/share/tensor/ctensor.mac

(%i19) `init_ctensor();`

(%o19) *done*

(%i20) `ct_coords: [t,x,y,z];`

(%o20) *[t, x, y, z]*

(%i21) `lg:ratsimp(g_ik);`

(%o21)
$$\begin{bmatrix} 1-U & 0 & 0 & 0 \\ 0 & \frac{(ax^2-1)\,U+1}{U-1} & \frac{ax\,ay\,U}{U-1} & \frac{ax\,az\,U}{U-1} \\ 0 & \frac{ax\,ay\,U}{U-1} & \frac{(ay^2-1)\,U+1}{U-1} & \frac{ay\,az\,U}{U-1} \\ 0 & \frac{ax\,az\,U}{U-1} & \frac{ay\,az\,U}{U-1} & \frac{(az^2-1)\,U+1}{U-1} \end{bmatrix}$$

(%i22) `cmetric();`

(%o22) *done*

(%i23) `ug:ratsimp(ug);`

(%o23)
$$\begin{bmatrix} -\frac{1}{U-1} & 0 & 0 & 0 \\ 0 & -\frac{(az^2+ay^2-1)\,U+1}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ax\,ay\,U}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ax\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} \\ 0 & \frac{ax\,ay\,U}{(az^2+ay^2+ax^2-1)\,U+1} & -\frac{(az^2+ax^2-1)\,U+1}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ay\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} \\ 0 & \frac{ax\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} & \frac{ay\,az\,U}{(az^2+ay^2+ax^2-1)\,U+1} & -\frac{(ay^2+ax^2-1)\,U+1}{(az^2+ay^2+ax^2-1)\,U+1} \end{bmatrix}$$

(%i24) `ratchristof;`

(%o24) *true*

(%i25) christof(all);

(%t25) $$lcs_{1,1,2}=\frac{U_x}{2}$$

(%t26) $$lcs_{1,1,3}=\frac{U_y}{2}$$

(%t27) $$lcs_{1,1,4}=\frac{U_z}{2}$$

(%t28) $$lcs_{1,2,1}=-\frac{U_x}{2}$$

(%t29) $$lcs_{1,3,1}=-\frac{U_y}{2}$$

(%t30) $$lcs_{1,4,1}=-\frac{U_z}{2}$$

(%t31) $$lcs_{2,2,2}=\frac{\frac{(ax^2-1)(U_x)+2\,ax(ax_x)U}{U-1}-\frac{((ax^2-1)U+1)(U_x)}{(U-1)^2}}{2}$$

(%t32) $$lcs_{2,2,3}=(-\frac{(ax^2-1)(U_y)+2\,ax(ax_y)U}{U-1}+\frac{((ax^2-1)U+1)(U_y)}{(U-1)^2}-\frac{2\,ax\,ay\,U(U_x)}{(U-1)^2}+\frac{2\,ax\,ay(U_x)}{U-1}+\frac{2\,ax(ay_x)U}{U-1}+\frac{2(ax_x)\,ay\,U}{U-1})/2$$

(%t33) $$lcs_{2,2,4}=(-\frac{(ax^2-1)(U_z)+2\,ax(ax_z)U}{U-1}+\frac{((ax^2-1)U+1)(U_z)}{(U-1)^2}-\frac{2\,ax\,az\,U(U_x)}{(U-1)^2}+\frac{2\,ax\,az(U_x)}{U-1}+\frac{2\,ax(az_x)U}{U-1}+\frac{2(ax_x)\,az\,U}{U-1})/2$$

(%t34) $$lcs_{2,3,2}=\frac{\frac{(ax^2-1)(U_y)+2\,ax(ax_y)U}{U-1}-\frac{((ax^2-1)U+1)(U_y)}{(U-1)^2}}{2}$$

(%t35) $$lcs_{2,3,3}=\frac{\frac{(ay^2-1)(U_x)+2\,ay(ay_x)U}{U-1}-\frac{((ay^2-1)U+1)(U_x)}{(U-1)^2}}{2}$$

(%t36) $$lcs_{2,3,4}=(\frac{ax\,ay\,U(U_z)}{(U-1)^2}-\frac{ax\,ay(U_z)}{U-1}-\frac{ax\,az\,U(U_y)}{(U-1)^2}+\frac{ax\,az(U_y)}{U-1}-\frac{ay\,az\,U(U_x)}{(U-1)^2}+\frac{ay\,az(U_x)}{U-1}+\frac{ay(az_x)U}{U-1}+\frac{(ay_x)\,az\,U}{U-1}+\frac{(ax_y)\,az\,U}{U-1}-\frac{ax(ay_z)U}{U-1}-\frac{(ax_z)\,ay\,U}{U-1})/2$$

(%t37) $$lcs_{2,4,2}=\frac{\frac{(ax^2-1)(U_z)+2\,ax(ax_z)U}{U-1}-\frac{((ax^2-1)U+1)(U_z)}{(U-1)^2}}{2}$$

(%t38) $$lcs_{2,4,3}=(-\frac{ax\,ay\,U(U_z)}{(U-1)^2}+\frac{ax\,ay(U_z)}{U-1}+\frac{ax\,az\,U(U_y)}{(U-1)^2}-\frac{ax\,az(U_y)}{U-1}-\frac{ay\,az\,U(U_x)}{(U-1)^2}+\frac{ay\,az(U_x)}{U-1}+\frac{ay(az_x)U}{U-1}+\frac{(ay_x)\,az\,U}{U-1}-\frac{(ax_y)\,az\,U}{U-1}+\frac{ax(ay_z)U}{U-1}+\frac{(ax_z)\,ay\,U}{U-1})/2$$

(%t39) $$lcs_{2,4,4}=\frac{\frac{(az^2-1)(U_x)+2\,az(az_x)U}{U-1}-\frac{((az^2-1)U+1)(U_x)}{(U-1)^2}}{2}$$

$$(\frac{2\,ax\,ay\,U(U_y)}{(U-1)^2}-\frac{2\,ax\,ay(U_y)}{U-1}-\frac{(ay^2-1)(U_x)+2\,ay(ay_x)U}$$

(%i73)
```
lcs[2,2,2];
```

(%o73)
$$\frac{\frac{(ax^2-1)(U_x)+2\,ax(ax_x)\,U}{U-1}-\frac{((ax^2-1)\,U+1)(U_x)}{(U-1)^2}}{2}$$

(%i74)
```
ratsimp(lcs[2,2,2]);
```

(%o74)
$$-\frac{ax^2(U_x)-2\,ax(ax_x)\,U^2+2\,ax(ax_x)\,U}{2\,U^2-4\,U+2}$$

(%i75)
```
riemann(false);
```

(%o75) *done*

(%i76)
```
ricci(false);
```

(%o76) *done*

(%i77)
```
mat_ricci: ratsimp( matrix(
        [ ric[1,1],ric[1,2],ric[1,3],ric[1,4]],
        [ ric[2,1],ric[2,2],ric[2,3],ric[2,4]],
        [ ric[3,1],ric[3,2],ric[3,3],ric[3,4]],
        [ ric[4,1],ric[4,2],ric[4,3],ric[4,4]]))$
```

(%i78)
```
forsezeri: [ratsimp(ric[2,1]),ratsimp(ric[3,1]),ratsimp(ric[4,1]),
        ratsimp(ric[1,2]),
        ratsimp(ric[1,3]),
        ratsimp(ric[1,4])];
```

(%o78) $[0,0,0,0,0,0]$

(%i79)
```
certozeri: [ratsimp(ric[2,3]-ric[3,2]), ratsimp(ric[2,4]-ric[4,2]),
        ratsimp(ric[3,4]-ric[4,3]) ];
```

(%o79) $[0,0,0]$

```
(%i80) ratsimp(ric[1,1]);
```

(%o80)
$$-\left(\left(\left(\left(2\,ay^2+2\,ax^2-2\right)az^2+2\,ay^4+\left(4\,ax^2-4\right)ay^2+2\,ax^4-4\,ax^2+2\right)U^3+\left(\left(-2\,ay^2-2\,ax^2+4\right)az^2-2\,ay^4+\left(8-4\,ax^2\right)ay^2-2\,ax^4+8\,ax^2-6\right)U^2+\left(-2\,az^2-4\,ay^2-4\,ax^2+6\right)U-2\right)\left(U_{zz}\right)+\left(\left(-ay^2-ax^2+1\right)az^2-ay^4+\left(2-2\,ax^2\right)ay^2-ax^4+2\,ax^2-1\right)U^2+\left(\left(-ay^2-ax^2-2\right)az^2-ay^4+\left(-2\,ax^2-1\right)ay^2-ax^4-ax^2+2\right)U+az^2-ay^2-ax^2-1\right)\left(U_z\right)^2+\left(\left(\left(2\,ay\,az^3+\left(2\,ay^3+\left(2\,ax^2-2\right)ay\right)az\right)U^2+\left(2\,ay\,az^3+\left(2\,ay^3+\left(2\,ax^2-2\right)ay\right)az\right)U+4\,ay\,az\right)\left(U_y\right)+\left(\left(2\,ax\,az^3+\left(2\,ax\,ay^2+2\,ax^3-2\,ax\right)az\right)U^2+\left(2\,ax\,az^3+\left(2\,ax\,ay^2+2\,ax^3-2\,ax\right)az\right)U+4\,ax\,az\right)\left(U_x\right)+\left(\left(-2\,ay^2-2\,ax^2+2\right)az\left(az_z\right)+\left(-2\,ax\,ay^2-2\,ax^3+2\,ax\right)\left(az_x\right)+\left(-2\left(ay_y\right)-2\left(ax_x\right)\right)az^3+\left(4\,ay\left(ay_z\right)+4\,ax\left(ax_z\right)\right)az^2+\left(\left(2-2\,ax^2\right)\left(ay_y\right)+2\,ax\,ay\left(ay_x\right)-2\left(ax_x\right)ay^2+2\,ax\left(ax_y\right)ay+2\left(ax_x\right)\right)az+\left(2\,ay^3+\left(2\,ax^2-2\right)ay\right)\left(ay_z\right)+2\,ax\left(ax_z\right)ay^2+\left(2\,ax^3-2\,ax\right)\left(ax_z\right)\right)U^3+\left(\left(2\,ay^2+2\,ax^2-4\right)az\left(az_z\right)+\left(2\,ax\,ay^2+2\,ax^3-4\,ax\right)\left(az_x\right)+\left(2\left(ay_y\right)+2\left(ax_x\right)\right)az^3+\left(-4\,ay\left(ay_z\right)-4\,ax\left(ax_z\right)\right)az^2+\left(\left(2\,ax^2-4\right)\left(ay_y\right)-2\,ax\,ay\left(ay_x\right)+2\left(ax_x\right)ay^2-2\,ax\left(ax_y\right)ay-4\left(ax_x\right)\right)az+\left(\left(4-2\,ax^2\right)ay-2\,ay^3\right)\left(ay_z\right)-2\,ax\left(ax_z\right)ay^2+\left(4\,ax-2\,ax^3\right)\left(ax_z\right)\right)U^2+\left(2\,az\left(az_z\right)+2\,ax\left(az_x\right)+\left(2\left(ay_y\right)+2\left(ax_x\right)\right)az-2\,ay\left(ay_z\right)-2\,ax\left(ax_z\right)\right)U\right)\left(U_z\right)+\left(\left(2\,az^4+\left(2\,ay^2+4\,ax^2-4\right)az^2+\left(2\,ax^2-2\right)ay^2+2\,ax^4-4\,ax^2+2\right)U^3+\left(-2\,az^4+\left(-2\,ay^2-4\,ax^2+8\right)az^2+\left(4-2\,ax^2\right)ay^2-2\,ax^4+8\,ax^2-6\right)U^2+\left(-4\,az^2-2\,ay^2-4\,ax^2+6\right)U-2\right)\left(U_{yy}\right)+\left(\left(\left(\left(4-4\,ax^2\right)ay-4\,ay^3\right)az-4\,ay\,az^3\right)U^3+\left(4\,ay\,az^3+\left(4\,ay^3+\left(4\,ax^2-8\right)ay\right)az\right)U^2+4\,ay\,az\,U\right)\left(U_{yz}\right)+\left(\left(-az^4+\left(-ay^2-2\,ax^2+2\right)az^2+\left(1-ax^2\right)ay^2-ax^4+2\,ax^2-1\right)U^2+\left(-az^4+\left(-ay^2-2\,ax^2-1\right)az^2+\left(-ax^2-2\right)ay^2-ax^4-ax^2+2\right)U-az^2+ay^2-ax^2-1\right)\left(U_y\right)^2+\left(\left(\left(2\,ax\,ay\,az^2+2\,ax\,ay^3+\left(2\,ax^3-2\,ax\right)ay\right)U^2+\left(2\,ax\,ay\,az^2+2\,ax\,ay^3+\left(2\,ax^3-2\,ax\right)ay\right)U+4\,ax\,ay\right)\left(U_x\right)+\left(\left(\left(2-2\,ax^2\right)ay-2\,ay^3\right)\left(az_z\right)+2\,ax\,ay\,az\left(az_x\right)-2\left(ay_z\right)az^3+\left(-2\,ay\left(ay_y\right)-2\,ax\left(ay_x\right)-2\left(ax_x\right)ay+2\,ax\left(ax_y\right)\right)az^2+\left(\left(2-2\,ax^2\right)\left(ay_z\right)+2\,ax\left(ax_z\right)ay\right)az+\left(2-2\,ax^2\right)ay\left(ay_y\right)+\left(2\,ax-2\,ax^3\right)\left(ay_x\right)-2\left(ax_x\right)ay^3+4\,ax\left(ax_y\right)ay^2+2\left(ax_x\right)ay+\left(2\,ax^3-2\,ax\right)\left(ax_y\right)\right)U^3+\left(\left(2\,ay^3+\left(2\,ax^2-4\right)ay\right)\left(az_z\right)-2\,ax\,ay\,az\left(az_x\right)+2\left(ay_z\right)az^3+\left(2\,ay\left(ay_y\right)+2\,ax\left(ay_x\right)+2\left(ax_x\right)ay-2\,ax\left(ax_y\right)\right)az^2+\left(\left(2\,ax^2-4\right)\left(ay_z\right)-2\,ax\left(ax_z\right)ay\right)az+\left(2\,ax^2-4\right)ay\left(ay_y\right)+\left(2\,ax^3-4\,ax\right)\left(ay_x\right)+2\left(ax_x\right)ay^3-4\,ax\left(ax_y\right)ay^2-4\left(ax_x\right)ay+\left(4\,ax-2\,ax^3\right)\left(ax_y\right)\right)U^2+\left(2\,ay\left(az_z\right)+2\left(ay_z\right)az+2\,ay\left(ay_y\right)+2\,ax\left(ay_x\right)+2\left(ax_x\right)ay-2\,ax\left(ax_y\right)\right)U\right)\left(U_y\right)+\left(\left(2\,az^4+\left(4\,ay^2+2\,ax^2-4\right)az^2+2\,ay^4+\left(2\,ax^2-4\right)ay^2-2\,ax^2+2\right)U^3+\left(-2\,az^4+\left(-4\,ay^2-2\,ax^2+8\right)az^2-2\,ay^4+\left(8-2\,ax^2\right)ay^2+4\,ax^2-6\right)U^2+\left(-4\,az^2-4\,ay^2-2\,ax^2+6\right)U-2\right)\left(U_{xx}\right)+\left(\left(\left(-4\,ax\,ay^2-4\,ax^3+4\,ax\right)az-4\,ax\,az^3\right)U^3+
```
```

```
(%i81) ratsimp(ric[2,2]);
```
<< Espressione troppo lunga da visualizzare! >>

```
(%i82) rq:x^2+y^2+z^2;
```
$$(\%o82)\quad z^2+y^2+x^2$$

```
(%i83) v0n:2*m/rq^(1/2);
```
$$(\%o83)\quad \frac{2\,m}{\sqrt{z^2+y^2+x^2}}$$

Specifico tutte le derivate di primo e secondo ordine.

```
(%i84) vxn:ratsimp(diff(v0n,x));
```
$$(\%o84)\quad -\frac{2\,m\,x}{\left(z^2+y^2+x^2\right)^{3/2}}$$

```
(%i85) vyn:ratsimp(diff(v0n,y))$
```

```
(%i86) vzn:ratsimp(diff(v0n,z))$
```

```
(%i87) vxxn:ratsimp(diff(vxn,x));
```
$$(\%o87)\quad -\frac{\sqrt{z^2+y^2+x^2}\left(2\,m\,z^2+2\,m\,y^2-4\,m\,x^2\right)}{z^6+\left(3\,y^2+3\,x^2\right)z^4+\left(3\,y^4+6\,x^2\,y^2+3\,x^4\right)z^2+y^6+3\,x^2\,y^4+3\,x^4\,y^2+x^6}$$

```
(%i88) vyyn:ratsimp(diff(vyn,y))$
```

```
(%i89) vzzn:ratsimp(diff(vzn,z))$
```

```
(%i90) vxyn:ratsimp(diff(vxn,y));
```
$$(\%o90)\quad \frac{6\,m\,x\,y}{\sqrt{z^2+y^2+x^2}\left(z^4+\left(2\,y^2+2\,x^2\right)z^2+y^4+2\,x^2\,y^2+x^4\right)}$$

```
(%i91) vxzn:ratsimp(diff(vxn,z))$
```

```
(%i92) vyzn:ratsimp(diff(vyn,z))$
```

Ora sostituisco alle derivate alcuni simboli ossia uso v0, vx, vy, vz, vxx, vxy, vzz, vxy, vxz, vyz.
Inizio la procedura con ric[1,1]

```
(%i93) rics:ratsimp(ric[2,2]);
```
<< Espressione troppo lunga da visualizzare! >>

```
(%i94) ric1:ratsimp(subst(vx,diff(U,x),rics))$
```

```
(%i95) ric2:ratsimp(subst(vy,diff(U,y),ric1))$
```

```
(%i96) ric3:ratsimp(subst(vz,diff(U,z),ric2))$

(%i97) ric4:ratsimp(subst(vxx,diff(U,x,2),ric3))$

(%i98) ric4xxy:ratsimp(subst(vxxy,diff(diff(U,x,2),y),ric4))$

(%i99) ric4xxz:ratsimp(subst(vxxz,diff(diff(U,x,2),z),ric4xxy))$

(%i100) ric4x:ratsimp(subst(vxxx,diff(U,x,3),ric4xxz))$

(%i101) ric5:ratsimp(subst(vyy,diff(U,y,2),ric4x))$

(%i102) ric5yyz:ratsimp(subst(vyyz,diff(diff(U,y,2),z),ric5))$

(%i103) ric5y:ratsimp(subst(vyyy,diff(U,y,3),ric5yyz))$

(%i104) ric6:ratsimp(subst(vzz,diff(U,z,2),ric5y))$

(%i105) ric6z:ratsimp(subst(vzzz,diff(U,z,3),ric6))$

(%i106) ric7:ratsimp(subst(vxy,diff(diff(U,x),y),ric6z))$

(%i107) ric7xyz:ratsimp(subst(vxyz,diff(diff(diff(U,x),y),z),ric7))$

(%i108) ric7xyy:ratsimp(subst(vxyy,diff(diff(U,x),y,2),ric7xyz))$

(%i109) ric8:ratsimp(subst(vxz,diff(diff(U,x),z),ric7xyy))$

(%i110) ric8xzz:ratsimp(subst(vxzz,diff(diff(U,x),z,2),ric8))$

(%i111) ric9:ratsimp(subst(vyz,diff(diff(U,y),z),ric8xzz))$

(%i112) ric9yzz:ratsimp(subst(vyzz,diff(diff(U,y),z,2),ric9))$

(%i113) rica:ratsimp(subst(v0,U,ric9yzz));
<< Espressione troppo lunga da visualizzare! >>

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero
o cosa vien fuori ?
```