

Prove di calcolo tensoriale

Provo a realizzare un documento wxm per il calcolo di tutte le grandezze derivate da un assegnato tensore metrico

```
(%i1) a_1:17*x-31*y;
```

```
(%o1) 17 x-31 y
```

```
(%i2) A_1:23*x+7*y;
```

```
(%o2) 7 y+23 x
```

```
(%i3) a_1*A_1;
```

```
(%o3) (17 x-31 y)(7 y+23 x)
```

```
(%i4) v: [a_1, A_1];
```

```
(%o4) [17 x-31 y, 7 y+23 x]
```

```
(%i5) length(v);
```

```
(%o5) 2
```

```
(%i6) v[1];
```

```
(%o6) 17 x-31 y
```

```
(%i7) diff(v,x);
```

```
(%o7) [17, 23]
```

```
(%i8) g_00:matrix(
[x^3+2*y^2*x-z/(x^2+y^2),0,0,x*y*z],
[0,x*x+y*y,x*y,x*z],
[0,y*x,y*y+z*z,y*z],
[x*y*z,x*z,y*z,z*z+x*x] );
```

```
(%o8) [
- $\frac{z}{y^2+x^2}+2xy^2+x^3$     0    0    x y z
0    y2+x2    x y    x z
0    x y    z2+y2    y z
x y z    x z    y z    z2+x2
]
```

```
(%i9) matrixp(g_00);
```

```
(%o9) true
```

```
(%i10) g_00[1];
```

```
(%o10) [- $\frac{z}{y^2+x^2}+2xy^2+x^3$ , 0, 0, x y z]
```

```
(%i11) g_00[1,1];
```

```
(%o11) - $\frac{z}{y^2+x^2}+2xy^2+x^3$ 
```

```
(%i12) diff(g_00,x);
```

$$\begin{pmatrix} \frac{2xz}{(y^2+x^2)^2} + 2y^2 + 3x^2 & 0 & 0 & yz \\ 0 & 2xy & z & \\ 0 & y & 0 & 0 \\ yz & z & 0 & 2x \end{pmatrix}$$

```
(%o12)
```

```
(%i13) g_11:invert(g_00);
```

```
(%o13)
```

$$\frac{(y^2+x^2)((z^2+x^2)(z^2+y^2)-y^2z^2)+xz(xy^2z-xz(z^2+y^2))-xy(xy(z^2+x^2)-xyz^2)}{\left(-\frac{z}{y^2+x^2}+2xy^2+x^3\right)((y^2+x^2)((z^2+x^2)(z^2+y^2)-y^2z^2)+xz(xy^2z-xz(z^2+y^2))-xy(xy(z^2+x^2)-xyz^2))+x^2yz^2(z^2+y^2)-x^2y^3z^2}$$

$$\frac{x^2yz^2(z^2+y^2)-x^2y^3z^2}{\left(-\frac{z}{y^2+x^2}+2xy^2+x^3\right)((y^2+x^2)((z^2+x^2)(z^2+y^2)-y^2z^2)+xz(xy^2z-xz(z^2+y^2))-xy(xy(z^2+x^2)-xyz^2))+xy^2(y^2+x^2)z^2-x^3y^2z^2}$$

$$\frac{x^3y^3z-x^3y^2z^2}{\left(-\frac{z}{y^2+x^2}+2xy^2+x^3\right)((y^2+x^2)((z^2+x^2)(z^2+y^2)-y^2z^2)+xz(xy^2z-xz(z^2+y^2))-xy(xy(z^2+x^2)-xyz^2))+x^3y^3z-x^3y^2z^2}$$

```
(%i14) ratsimp(g_00.g_11);
```

```
(%o14)
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i15) ratsimp(g_11);
```

```
(%o15)
```

$$\frac{(y^4+x^2y^2)z^4+(x^2y^4+2x^4y^2+x^6)z^2+x^2y^6+x^4y^4}{y^2z^5+((x^2-2x)y^6+(2x^4-3x^3)y^4+(x^6-x^5)y^2)z^4+(x^2y^2+x^4)z^3+(x^2y^8+(x^4-2x^3)y^6-5x^5y^4-4x^7y^2-x^9)}$$

$$\frac{(x^2y^3+x^4y)z^4}{y^2z^5+((x^2-2x)y^6+(2x^4-3x^3)y^4+(x^6-x^5)y^2)z^4+(x^2y^2+x^4)z^3+(x^2y^8+(x^4-2x^3)y^6-5x^5y^4-4x^7y^2-x^9)}$$

$$\frac{(xy^6+x^3y^4)z^2}{y^2z^5+((x^2-2x)y^6+(2x^4-3x^3)y^4+(x^6-x^5)y^2)z^4+(x^2y^2+x^4)z^3+(x^2y^8+(x^4-2x^3)y^6-5x^5y^4-4x^7y^2-x^9)}$$

$$\frac{(xy^5+2x^3y^3+x^5y)z^3+(xy^7+x^3y^5)z}{y^2z^5+((x^2-2x)y^6+(2x^4-3x^3)y^4+(x^6-x^5)y^2)z^4+(x^2y^2+x^4)z^3+(x^2y^8+(x^4-2x^3)y^6-5x^5y^4-4x^7y^2-x^9)}$$

```
(%i16) v1: [ (x^2-y^2)/(x+y)^2, (x^3+y^3)/(x^2-y^2) ];
```

```
(%o16) [  $\frac{x^2-y^2}{(y+x)^2}$ ,  $\frac{y^3+x^3}{x^2-y^2}$  ]
```

```
(%i17) ratsimp(v1);
```

```
(%o17) [  $-\frac{y-x}{y+x}$ ,  $-\frac{y^2-xy+x^2}{y-x}$  ]
```