Prove di calcolo tensoriale

Provo a realizzare un documento wxm per il calcolo di tutte
le grandezze derivate da un assegnato tensore metrico.
In questo caso uso il tensore metrico di Reissner e Nordstrom.

```
(%i1) (if atom(lg) then load(ctensor));
```
(%o1)
C:/Programmi/Maxima-5.20.1/share/maxima/5.20.1/share/tensor/ctensor.mac

```
(%i2) init_ctensor();
```
(%o2) *done*

Definisco una funzione del tutto generica delle coordinate.

```
(%i3) rq:x^2+y^2+z^2;
```
(%o3) $z^2+y^2+x^2$

```
(%i4) depends(a,[x,y,z]);
```
(%o4) *[a(x, y, z)]*

La funzione b mi serve solo per semplificare l'input.

```
(%i5) b:a/(a-1);
```
(%o5) $\frac{a}{a-1}$

```
(%i6) lgmia: matrix (
      [ 1-a, 0,0,0 ],
      [ 0,-1+b*x^2/rq,b*x*y/rq, b*x*z/rq ],
      [ 0,b*x*y/rq,-1+b*y^2/rq,b*y*z/rq ],
      [ 0,b*x*z/rq,b*y*z/rq,-1+b*z^2/rq]);
```
(%o6)
$$\begin{bmatrix} 1-a & 0 & 0 & 0 \\ 0 & \frac{a\,x^2}{(a-1)(z^2+y^2+x^2)}-1 & \frac{a\,x\,y}{(a-1)(z^2+y^2+x^2)} & \frac{a\,x\,z}{(a-1)(z^2+y^2+x^2)} \\ 0 & \frac{a\,x\,y}{(a-1)(z^2+y^2+x^2)} & \frac{a\,y^2}{(a-1)(z^2+y^2+x^2)}-1 & \frac{a\,y\,z}{(a-1)(z^2+y^2+x^2)} \\ 0 & \frac{a\,x\,z}{(a-1)(z^2+y^2+x^2)} & \frac{a\,y\,z}{(a-1)(z^2+y^2+x^2)} & \frac{a\,z^2}{(a-1)(z^2+y^2+x^2)}-1 \end{bmatrix}$$

Dichiaro che faccio uso di coordinate cartesiane

```
(%i7) ct_coords: [t,x,y,z];
```
(%o7) *[t, x, y, z]*

In base alle esigenze della libreria ctensor di cui ho fatto il
load inizializzo il tensore metrico covariante che si deve chiamare lg.

(%i8) lg:ratsimp(lgmia);

(%o8)

$$\begin{bmatrix} 1-a & 0 & 0 & 0 \\ 0 & -\frac{(a-1)\,z^2+(a-1)\,y^2-x^2}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,x\,y}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,x\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} \\ 0 & \frac{a\,x\,y}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & -\frac{(a-1)\,z^2-y^2+(a-1)\,x^2}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,y\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} \\ 0 & \frac{a\,x\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,y\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{z^2+(1-a)\,y^2+(1-a)\,x^2}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} \end{bmatrix}$$

Calcolo la metrica ovvero il tensore metrico controvariante che deve essere la matrice inversa del tensore metrico covariante.

(%i9) cmetric();

(%o9) *done*

(%i10) uug:ug$

Cerco di semplificare il piu' possibile l'espressione del tensore metrico controvariante

(%i11) ug:ratsimp(uug);

(%o11)

$$\begin{bmatrix} -\frac{1}{a-1} & 0 & 0 & 0 \\ 0 & -\frac{z^2+y^2+(1-a)\,x^2}{z^2+y^2+x^2} & \frac{a\,x\,y}{z^2+y^2+x^2} & \frac{a\,x\,z}{z^2+y^2+x^2} \\ 0 & \frac{a\,x\,y}{z^2+y^2+x^2} & -\frac{z^2+(1-a)\,y^2+x^2}{z^2+y^2+x^2} & \frac{a\,y\,z}{z^2+y^2+x^2} \\ 0 & \frac{a\,x\,z}{z^2+y^2+x^2} & \frac{a\,y\,z}{z^2+y^2+x^2} & \frac{(a-1)\,z^2-y^2-x^2}{z^2+y^2+x^2} \end{bmatrix}$$

Anche se non serve ora faccio vedere che lg ed ug sono una la matrice inversa dell'altra

Ora calcolo i simboli di christoffel di prima e seconda specie visualizzandoli tutti. Attenzione alle regole della libreria ctensor che mette come terzo indice quello che di solito viene scritto come primo indice.

(%i12) christof(all);

(%t12) $lcs_{1,1,2}=\frac{a_x}{2}$

(%t13) $lcs_{1,1,3}=\frac{a_y}{2}$

(%t14) $lcs_{1,1,4}=\frac{a_z}{2}$

(%t15) $lcs_{1,2,1}=-\frac{a_x}{2}$

(%t16) $lcs_{1,3,1}=-\frac{a_y}{2}$

(%t17) $lcs_{1,4,1}=-\frac{a_z}{2}$

(%t18) $lcs_{2,2,2}=$

$$\frac{((a-1)z^2+(a-1)y^2-x^2)((a_x)z^2+(a_x)y^2+(a_x)x^2+2(a-1)x)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}-\frac{(a_x)z^2+(a_x)y^2-2x}{(a-1)z^2+(a-1)y^2+(a-1)x^2}$$
$$2$$

(%t19) $lcs_{2,2,3}=($

$$-\frac{((a-1)z^2+(a-1)y^2-x^2)((a_y)z^2+(a_y)y^2+2(a-1)y+(a_y)x^2)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}+$$
$$\frac{(a_y)z^2+(a_y)y^2+2(a-1)y}{(a-1)z^2+(a-1)y^2+(a-1)x^2}-\frac{2axy((a_x)z^2+(a_x)y^2+(a_x)x^2+2(a-1)x)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}+$$
$$\frac{2(a_x)xy}{(a-1)z^2+(a-1)y^2+(a-1)x^2}+\frac{2ay}{(a-1)z^2+(a-1)y^2+(a-1)x^2})/2$$

(%t20) $lcs_{2,2,4}=($

$$-\frac{((a-1)z^2+(a-1)y^2-x^2)((a_z)z^2+2(a-1)z+(a_z)y^2+(a_z)x^2)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}+$$
$$\frac{(a_z)z^2+2(a-1)z+(a_z)y^2}{(a-1)z^2+(a-1)y^2+(a-1)x^2}-\frac{2axz((a_x)z^2+(a_x)y^2+(a_x)x^2+2(a-1)x)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}+$$
$$\frac{2(a_x)xz}{(a-1)z^2+(a-1)y^2+(a-1)x^2}+\frac{2az}{(a-1)z^2+(a-1)y^2+(a-1)x^2})/2$$

(%t21) $lcs_{2,3,2}=$

$$\frac{((a-1)z^2+(a-1)y^2-x^2)((a_y)z^2+(a_y)y^2+2(a-1)y+(a_y)x^2)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}-\frac{(a_y)z^2+(a_y)y^2+2(a-1)y}{(a-1)z^2+(a-1)y^2+(a-1)x^2}$$
$$2$$

(%t22) $lcs_{2,3,3}=$

$$\frac{((a-1)z^2-y^2+(a-1)x^2)((a_x)z^2+(a_x)y^2+(a_x)x^2+2(a-1)x)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}-\frac{(a_x)z^2+(a_x)x^2+2(a-1)x}{(a-1)z^2+(a-1)y^2+(a-1)x^2}$$
$$2$$

(%t23) $lcs_{2,3,4}=($

$$\frac{axy((a_z)z^2+2(a-1)z+(a_z)y^2+(a_z)x^2)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}-$$
$$\frac{axz((a_y)z^2+(a_y)y^2+2(a-1)y+(a_y)x^2)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}-\frac{ayz((a_x)z^2+(a_x)y^2+(a_x)x^2+2(a-1)x)}{((a-1)z^2+(a-1)y^2+(a-1)x^2)^2}+$$
$$\frac{(a_x)yz}{(a-1)z^2+(a-1)y^2+(a-1)x^2}+\frac{(a_y)xz}{(a-1)z^2+(a-1)y^2+(a-1)x^2}-$$

Provo a semplificare una delle componenti del simbolo di Christoffel
di seconda specie.

```
(%i60) ratsimp(mcs[2,2,2]);
```

$$(\%o60)\ -(((2a-1)(a_x)x^2+(-2a^3+4a^2-2a)x)z^2-a(a_z)x^3z+$$
$$((2a-1)(a_x)x^2+(-2a^3+4a^2-2a)x)y^2-a(a_y)x^3y+(a-1)(a_x)x^4)/((2a^2-4a+2)$$
$$z^4+((4a^2-8a+4)y^2+(4a^2-8a+4)x^2)z^2+(2a^2-4a+2)y^4+(4a^2-8a+4)x^2y^2+$$
$$(2a^2-4a+2)x^4)$$

Creo il tensore di Riemann sopprimendo l'output che probabilmente sarebbe
troppo lungo per essere stampato.

```
(%i61) riemann(false);
(%o61) done
```

Ora faccio lo stesso calcolando il tensore di Ricci.

```
(%i62) ricci(false);
(%o62) done
```

Ora trasformo il tensore di Ricci in matrice.

```
(%i63) mat_ricci: ratsimp( matrix(
       [ ric[1,1],ric[1,2],ric[1,3],ric[1,4]],
       [ ric[2,1],ric[2,2],ric[2,3],ric[2,4]],
       [ ric[3,1],ric[3,2],ric[3,3],ric[3,4]],
       [ ric[4,1],ric[4,2],ric[4,3],ric[4,4]]))$
```

Innanzi tutto controllo che i termini non diagonali della
prima riga e colonna siano degli zeri.

```
(%i64) forsezeri: [ratsimp(ric[2,1]),ratsimp(ric[3,1]),ratsimp(ric[4,1]),
       ratsimp(ric[1,2]),
       ratsimp(ric[1,3]),
       ratsimp(ric[1,4])];
(%o64) [0,0,0,0,0,0]
```

Controllo che il tensore di ricci e' un tensore simmetrico... come da manua
ma se uno vuol fare il san Tommaso...

```
(%i65) certozeri: [ratsimp(ric[2,3]-ric[3,2]), ratsimp(ric[2,4]-ric[4,2]),
       ratsimp(ric[3,4]-ric[4,3]) ];
(%o65) [0,0,0]
```

Questi non sono zeri ma se la funzione a(x,y,z) assume
dei valori adeguati lo debbono diventare...
Intanto li stampo...

```
(%i66) ratsimp(ric[1,1]);
```

$$(%o66)\ \left(\left((a^2-2\,a+1)(a_{zz})+(1-a)(a_{yy})+(a_y)^2+(1-a)(a_{xx})+(a_x)^2\right)z^2+\left(\left((2\,a^2-2\,a)(a_{yz})-2\,(a_y)(a_z)\right)y+\left((2\,a^2-2\,a)(a_{xz})-2\,(a_x)(a_z)\right)x+(2\,a^2-2\,a)(a_z)\right)z+\left((1-a)(a_{zz})+(a_z)^2+(a^2-2\,a+1)(a_{yy})+(1-a)(a_{xx})+(a_x)^2\right)y^2+\left(\left((2\,a^2-2\,a)(a_{xy})-2\,(a_x)(a_y)\right)x+(2\,a^2-2\,a)(a_y)\right)y+\left((1-a)(a_{zz})+(a_z)^2+(1-a)(a_{yy})+(a_y)^2+(a^2-2\,a+1)(a_{xx})\right)x^2+(2\,a^2-2\,a)(a_x)\,x\right)/\left((2\,a-2)\,z^2+(2\,a-2)\,y^2+(2\,a-2)\,x^2\right)$$

```
(%i67) ratsimp(ric[2,2]);
```

$$(%o67)\ -\left((a-1)(a_x)^2\,z^4+\left(\left((2-2\,a)(a_x)(a_z)+(2\,a^2-4\,a+2)(a_{xz})\right)x+(-2\,a^3+6\,a^2-6\,a+2)(a_z)\right)z^3+\left((2\,a-2)(a_x)^2\,y^2+\left(\left((2-2\,a)(a_x)(a_y)+(2\,a^2-4\,a+2)(a_{xy})\right)x+(-2\,a^3+6\,a^2-6\,a+2)(a_y)\right)y+\left((-a^2+2\,a-1)(a_{zz})+(a-1)(a_z)^2+(a-1)(a_{yy})-(a_y)^2+(2\,a^2-3\,a+1)(a_{xx})-(a_x)^2\right)x^2+(-2\,a^3+8\,a^2-10\,a+4)(a_x)\,x-2\,a^4+6\,a^3-6\,a^2+2\,a\right)z^2+\left(\left(\left((2-2\,a)(a_x)(a_z)+(2\,a^2-4\,a+2)(a_{xz})\right)x+(-2\,a^3+6\,a^2-6\,a+2)(a_z)\right)y^2+\left(2\,a(a_y)(a_z)+(2\,a-2\,a^2)(a_{yz})\right)x^2\,y+(2\,(a_x)(a_z)+(2-2\,a)(a_{xz}))\,x^3+(2-2\,a)(a_z)\,x^2\right)z+(a-1)(a_x)^2\,y^4+\left(\left((2-2\,a)(a_x)(a_y)+(2\,a^2-4\,a+2)(a_{xy})\right)x+(-2\,a^3+6\,a^2-6\,a+2)(a_y)\right)y^3+\left((a-1)(a_{zz})-(a_z)^2+(-a^2+2\,a-1)(a_{yy})+(a-1)(a_y)^2+(2\,a^2-3\,a+1)(a_{xx})-(a_x)^2\right)x^2+(-2\,a^3+8\,a^2-10\,a+4)(a_x)\,x-2\,a^4+6\,a^3-6\,a^2+2\,a\right)y^2+\left(2\,(a_x)(a_y)+(2-2\,a)(a_{xy}))\,x^3+(2-2\,a)(a_y)\,x^2\right)y+\left((a-1)(a_{zz})-(a_z)^2+(a-1)(a_{yy})-(a_y)^2+(a^2-2\,a+1)(a_{xx})\right)x^4+(2\,a^2-6\,a+4)(a_x)\,x^3\right)/\left((2\,a^3-6\,a^2+6\,a-2)\,z^4+\left((4\,a^3-12\,a^2+12\,a-4)\,y^2+(4\,a^3-12\,a^2+12\,a-4)\,x^2\right)z^2+(2\,a^3-6\,a^2+6\,a-2)\,y^4+(4\,a^3-12\,a^2+12\,a-4)\,x^2\,y^2+(2\,a^3-6\,a^2+6\,a-2)\,x^4\right)$$

```
(%i68) ratsimp(ric[3,3]);
```

$$
\begin{aligned}
(\%o68)\ & -((a-1)(a_y)^2 z^4 + \\
& \left(\left((2-2a)(a_y)(a_z)+(2a^2-4a+2)(a_{yz})\right)y+(-2a^3+6a^2-6a+2)(a_z)\right)z^3+(\\
& \left((-a^2+2a-1)(a_{zz})+(a-1)(a_z)^2+(2a^2-3a+1)(a_{yy})-(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)y^2+ \\
& \left(\left((2-2a)(a_x)(a_y)+(2a^2-4a+2)(a_{xy})\right)x+(-2a^3+8a^2-10a+4)(a_y)\right)y+(2a-2) \\
& (a_y)^2 x^2+(-2a^3+6a^2-6a+2)(a_x)x-2a^4+6a^3-6a^2+2a)z^2+(\\
& (2(a_y)(a_z)+(2-2a)(a_{yz}))y^3+\left(\left(2a(a_x)(a_z)+(2a-2a^2)(a_{xz})\right)x+(2-2a)(a_z)\right)y^2+ \\
& \left((2-2a)(a_y)(a_z)+(2a^2-4a+2)(a_{yz})\right)x^2 y+(-2a^3+6a^2-6a+2)(a_z)x^2)z+ \\
& \left((a-1)(a_{zz})-(a_z)^2+(a^2-2a+1)(a_{yy})+(a-1)(a_{xx})-(a_x)^2\right)y^4+ \\
& \left((2(a_x)(a_y)+(2-2a)(a_{xy}))x+(2a^2-6a+4)(a_y)\right)y^3+(\\
& \left((a-1)(a_{zz})-(a_z)^2+(2a^2-3a+1)(a_{yy})-(a_y)^2+(-a^2+2a-1)(a_{xx})+(a-1)(a_x)^2\right)x^2+ \\
& (2-2a)(a_x)x)y^2+ \\
& \left(\left((2-2a)(a_x)(a_y)+(2a^2-4a+2)(a_{xy})\right)x^3+(-2a^3+8a^2-10a+4)(a_y)x^2\right)y+(a-1) \\
& (a_y)^2 x^4+(-2a^3+6a^2-6a+2)(a_x)x^3+(-2a^4+6a^3-6a^2+2a)x^2)/(\\
& (2a^3-6a^2+6a-2)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+ \\
& (2a^3-6a^2+6a-2)y^4+(4a^3-12a^2+12a-4)x^2 y^2+(2a^3-6a^2+6a-2)x^4)
\end{aligned}
$$

```
(%i69) ratsimp(ric[4,4]);
```

$$
\begin{aligned}
(\%o69)\ & -(\left((a^2-2a+1)(a_{zz})+(a-1)(a_{yy})-(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)z^4+ \\
& \left((2(a_y)(a_z)+(2-2a)(a_{yz}))y+(2(a_x)(a_z)+(2-2a)(a_{xz}))x+(2a^2-6a+4)(a_z)\right)z^3+(\\
& \left((2a^2-3a+1)(a_{zz})-(a_z)^2+(-a^2+2a-1)(a_{yy})+(a-1)(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)y^2+ \\
& \left(\left(2a(a_x)(a_y)+(2a-2a^2)(a_{xy})\right)x+(2-2a)(a_y)\right)y+ \\
& \left((2a^2-3a+1)(a_{zz})-(a_z)^2+(a-1)(a_{yy})-(a_y)^2+(-a^2+2a-1)(a_{xx})+(a-1)(a_x)^2\right)x^2+ \\
& (2-2a)(a_x)x)z^2+(\left((2-2a)(a_y)(a_z)+(2a^2-4a+2)(a_{yz})\right)y^3+ \\
& \left(\left((2-2a)(a_x)(a_z)+(2a^2-4a+2)(a_{xz})\right)x+(-2a^3+8a^2-10a+4)(a_z)\right)y^2+ \\
& \left((2-2a)(a_y)(a_z)+(2a^2-4a+2)(a_{yz})\right)x^2 y+\left((2-2a)(a_x)(a_z)+(2a^2-4a+2)(a_{xz})\right) \\
& x^3+(-2a^3+8a^2-10a+4)(a_z)x^2)z+(a-1)(a_z)^2 y^4+(-2a^3+6a^2-6a+2)(a_y)y^3+ \\
& \left((2a-2)(a_z)^2 x^2+(-2a^3+6a^2-6a+2)(a_x)x-2a^4+6a^3-6a^2+2a\right)y^2+ \\
& (-2a^3+6a^2-6a+2)(a_y)x^2 y+(a-1)(a_z)^2 x^4+(-2a^3+6a^2-6a+2)(a_x)x^3+ \\
& (-2a^4+6a^3-6a^2+2a)x^2)/((2a^3-6a^2+6a-2)z^4+ \\
& ((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+(2a^3-6a^2+6a-2)y^4+ \\
& (4a^3-12a^2+12a-4)x^2 y^2+(2a^3-6a^2+6a-2)x^4)
\end{aligned}
$$

```
(%i70) ratsimp(ric[2,3]);
```

$$
\begin{aligned}
(\%o70)\ & -((a-1)(a_x)(a_y)z^4+\\
&\left(\left((1-a)(a_x)(a_z)+(a^2-2a+1)(a_{xz})\right)y+\left((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz})\right)x\right)z^3+(\\
&\left((a-1)(a_x)(a_y)+(a^2-2a+1)(a_{xy})\right)y^2+(\\
&\left((-a^2+2a-1)(a_{zz})+(a-1)(a_z)^2+(a^2-a)(a_{yy})-a(a_y)^2+(a^2-a)(a_{xx})-a(a_x)^2\right)x+\\
&(a^2-2a+1)(a_x))\,y+\left((a-1)(a_x)(a_y)+(a^2-2a+1)(a_{xy})\right)x^2+(a^2-2a+1)(a_y)x)\,z^2+\\
&(\left((1-a)(a_x)(a_z)+(a^2-2a+1)(a_{xz})\right)y^3+\left((a+1)(a_y)(a_z)+(1-a^2)(a_{yz})\right)xy^2+\\
&\left(\left((a+1)(a_x)(a_z)+(1-a^2)(a_{xz})\right)x^2+(2a^3-6a^2+4a)(a_z)x\right)y+\\
&\left((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz})\right)x^3)\,z+(a^2-2a+1)(a_{xy})y^4+\\
&\left(\left((a-1)(a_{zz})-(a_z)^2+(a^2-a)(a_{xx})-a(a_x)^2\right)x+(a^2-2a+1)(a_x)\right)y^3+\\
&\left((2a(a_x)(a_y)+(2-2a)(a_{xy}))x^2+(2a^3-5a^2+2a+1)(a_y)x\right)y^2+(\\
&\left((a-1)(a_{zz})-(a_z)^2+(a^2-a)(a_{yy})-a(a_y)^2\right)x^3+(2a^3-5a^2+2a+1)(a_x)x^2+\\
&(2a^4-6a^3+6a^2-2a)x)\,y+(a^2-2a+1)(a_{xy})x^4+(a^2-2a+1)(a_y)x^3)/(\\
&(2a^3-6a^2+6a-2)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+\\
&(2a^3-6a^2+6a-2)y^4+(4a^3-12a^2+12a-4)x^2y^2+(2a^3-6a^2+6a-2)x^4)
\end{aligned}
$$

```
(%i71) ratsimp(ric[2,4]);
```

$$
\begin{aligned}
(\%o71)\ & -((a^2-2a+1)(a_{xz})z^4+(\left((1-a)(a_x)(a_y)+(a^2-2a+1)(a_{xy})\right)y+\\
&\left((a-1)(a_{yy})-(a_y)^2+(a^2-a)(a_{xx})-a(a_x)^2\right)x+(a^2-2a+1)(a_x))\,z^3+(\\
&\left((a-1)(a_x)(a_z)+(a^2-2a+1)(a_{xz})\right)y^2+\left((a+1)(a_y)(a_z)+(1-a^2)(a_{yz})\right)xy+\\
&(2a(a_x)(a_z)+(2-2a)(a_{xz}))x^2+(2a^3-5a^2+2a+1)(a_z)x)\,z^2+(\\
&\left((1-a)(a_x)(a_y)+(a^2-2a+1)(a_{xy})\right)y^3+(\\
&\left((a^2-a)(a_{zz})-a(a_z)^2+(-a^2+2a-1)(a_{yy})+(a-1)(a_y)^2+(a^2-a)(a_{xx})-a(a_x)^2\right)x+\\
&(a^2-2a+1)(a_x))\,y^2+\left(\left((a+1)(a_x)(a_y)+(1-a^2)(a_{xy})\right)x^2+(2a^3-6a^2+4a)(a_y)x\right)y+\\
&\left((a^2-a)(a_{zz})-a(a_z)^2+(a-1)(a_{yy})-(a_y)^2\right)x^3+(2a^3-5a^2+2a+1)(a_x)x^2+\\
&(2a^4-6a^3+6a^2-2a)x)\,z+(a-1)(a_x)(a_z)y^4+\left((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz})\right)x\\
&y^3+\left(\left((a-1)(a_x)(a_z)+(a^2-2a+1)(a_{xz})\right)x^2+(a^2-2a+1)(a_z)x\right)y^2+\\
&\left((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz})\right)x^3\,y+(a^2-2a+1)(a_{xz})x^4+(a^2-2a+1)(a_z)x^3)/\\
&((2a^3-6a^2+6a-2)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+\\
&(2a^3-6a^2+6a-2)y^4+(4a^3-12a^2+12a-4)x^2y^2+(2a^3-6a^2+6a-2)x^4)
\end{aligned}
$$

(%i72) ratsimp(ric[3,4]);

(%o72) $$-((a^2-2\,a+1)(a_{yz})\,z^4+(((a^2-a)(a_{yy})-a(a_y)^2+(a-1)(a_{xx})-(a_x)^2)\,y+((1-a)(a_x)(a_y)+(a^2-2\,a+1)(a_{xy}))\,x+(a^2-2\,a+1)(a_y))\,z^3+(((a+1)(a_x)(a_z)+(1-a^2)(a_{xz}))\,x+(2\,a^3-5\,a^2+2\,a+1)(a_z))\,y+((a-1)(a_y)(a_z)+(a^2-2\,a+1)(a_{yz}))\,x^2)\,z^2+(((a^2-a)(a_{zz})-a(a_z)^2+(a-1)(a_{xx})-(a_x)^2)\,y^3+(((a+1)(a_x)(a_y)+(1-a^2)(a_{xy}))\,x+(2\,a^3-5\,a^2+2\,a+1)(a_y))\,y^2+((a^2-a)(a_{zz})-a(a_z)^2+(a^2-a)(a_{yy})-a(a_y)^2+(-a^2+2\,a-1)(a_{xx})+(a-1)(a_x)^2)\,x^2+(2\,a^3-6\,a^2+4\,a)(a_x)\,x+2\,a^4-6\,a^3+6\,a^2-2\,a)\,y+((1-a)(a_x)(a_y)+(a^2-2\,a+1)(a_{xy}))\,x^3+(a^2-2\,a+1)(a_y)\,x^2)\,z+(a^2-2\,a+1)(a_{yz})\,y^4+(((1-a)(a_x)(a_z)+(a^2-2\,a+1)(a_{xz}))\,x+(a^2-2\,a+1)(a_z))\,y^3+((a-1)(a_y)(a_z)+(a^2-2\,a+1)(a_{yz}))\,x^2\,y^2+(((1-a)(a_x)(a_z)+(a^2-2\,a+1)(a_{xz}))\,x^3+(a^2-2\,a+1)(a_z)\,x^2)\,y+(a-1)(a_y)(a_z)\,x^4)/((2\,a^3-6\,a^2+6\,a-2)\,z^4+((4\,a^3-12\,a^2+12\,a-4)\,y^2+(4\,a^3-12\,a^2+12\,a-4)\,x^2)\,z^2+(2\,a^3-6\,a^2+6\,a-2)\,y^4+(4\,a^3-12\,a^2+12\,a-4)\,x^2\,y^2+(2\,a^3-6\,a^2+6\,a-2)\,x^4)$$

Ora mi preparo a specificare cosa deve valere la funzione a(x,y,z) per fare in modo che la metrica sia quella di un buco nero neutro ossia sia la metri di Schwarzschild.

(%i73) v0n:2*m/rq^(1/2);

(%o73) $$\frac{2\,m}{\sqrt{z^2+y^2+x^2}}$$

Specifico tutte le derivate di primo e secondo ordine.

(%i74) vxn:ratsimp(diff(v0n,x));

(%o74) $$-\frac{2\,m\,x}{(z^2+y^2+x^2)^{3/2}}$$

(%i75) vyn:ratsimp(diff(v0n,y))$

(%i76) vzn:ratsimp(diff(v0n,z))$

(%i77) vxxn:ratsimp(diff(vxn,x));

(%o77) $$-\frac{\sqrt{z^2+y^2+x^2}\,(2\,m\,z^2+2\,m\,y^2-4\,m\,x^2)}{z^6+(3\,y^2+3\,x^2)\,z^4+(3\,y^4+6\,x^2\,y^2+3\,x^4)\,z^2+y^6+3\,x^2\,y^4+3\,x^4\,y^2+x^6}$$

(%i78) vyyn:ratsimp(diff(vyn,y))$

```
(%i79) vzzn:ratsimp(diff(vzn,z))$
```

```
(%i80) vxyn:ratsimp(diff(vxn,y));
```

$$(\%o80)\quad \frac{6\,m\,x\,y}{\sqrt{z^2+y^2+x^2}\,(z^4+(2\,y^2+2\,x^2)\,z^2+y^4+2\,x^2\,y^2+x^4)}$$

```
(%i81) vxzn:ratsimp(diff(vxn,z))$
```

```
(%i82) vyzn:ratsimp(diff(vyn,z))$
```

Ora sostituisco alle derivate alcuni simboli ossia uso v0, vx, vy, vz, vxx, vxy, vzz, vxy, vxz, vyz.
Inizio la procedura con ric[1,1]

```
(%i83) rics:ratsimp(ric[1,1]);
```

$$(\%o83)\quad (((a^2-2\,a+1)(a_{zz})+(1-a)(a_{yy})+(a_y)^2+(1-a)(a_{xx})+(a_x)^2)\,z^2+$$
$$(((2\,a^2-2\,a)(a_{yz})-2\,(a_y)(a_z))\,y+((2\,a^2-2\,a)(a_{xz})-2\,(a_x)(a_z))\,x+(2\,a^2-2\,a)(a_z))\,z+$$
$$((1-a)(a_{zz})+(a_z)^2+(a^2-2\,a+1)(a_{yy})+(1-a)(a_{xx})+(a_x)^2)\,y^2+$$
$$(((2\,a^2-2\,a)(a_{xy})-2\,(a_x)(a_y))\,x+(2\,a^2-2\,a)(a_y))\,y+$$
$$((1-a)(a_{zz})+(a_z)^2+(1-a)(a_{yy})+(a_y)^2+(a^2-2\,a+1)(a_{xx}))\,x^2+(2\,a^2-2\,a)(a_x)\,x)/($$
$$(2\,a-2)\,z^2+(2\,a-2)\,y^2+(2\,a-2)\,x^2)$$

```
(%i84) ric1:ratsimp(subst(vx,diff(a,x),rics))$
```

```
(%i85) ric2:ratsimp(subst(vy,diff(a,y),ric1))$
```

```
(%i86) ric3:ratsimp(subst(vz,diff(a,z),ric2))$
```

```
(%i87) ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$
```

```
(%i88) ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$
```

```
(%i89) ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$
```

```
(%i90) ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$
```

```
(%i91) ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$
```

```
(%i92) ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

```
(%i93) rica:ratsimp(subst(v0,a,ric9));
```

$$(\%o93)\ (((v0^2-2\,v0+1)\,vzz+(1-v0)\,vyy+vy^2+(1-v0)\,vxx+vx^2)\,z^2+(((2\,v0^2-2\,v0)\,vyz-2\,vy\,vz)\,y+((2\,v0^2-2\,v0)\,vxz-2\,vx\,vz)\,x+(2\,v0^2-2\,v0)\,vz)\,z+((1-v0)\,vzz+vz^2+(v0^2-2\,v0+1)\,vyy+(1-v0)\,vxx+vx^2)\,y^2+(((2\,v0^2-2\,v0)\,vxy-2\,vx\,vy)\,x+(2\,v0^2-2\,v0)\,vy)\,y+((1-v0)\,vzz+vz^2+(1-v0)\,vyy+vy^2+(v0^2-2\,v0+1)\,vxx)\,x^2+(2\,v0^2-2\,v0)\,vx\,x)/(2\,v0-2)\,z^2+(2\,v0-2)\,y^2+(2\,v0-2)\,x^2)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i94) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i95) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i96) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i97) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i98) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i99) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i100) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i101) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i102) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i103) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i104) ratsimp(rican);
(%o104) 0
```

Ripeto tutta la procedura con ric[2,2]

```
(%i105) rics:ratsimp(ric[2,2]);
```

(%o105)

$$-((a-1)(a_x)^2 z^4+\left(\left((2-2a)(a_x)(a_z)+(2a^2-4a+2)(a_{xz})\right)x+(-2a^3+6a^2-6a+2)(a_z)\right)z^3+((2a-2)(a_x)^2 y^2+\left(\left((2-2a)(a_x)(a_y)+(2a^2-4a+2)(a_{xy})\right)x+(-2a^3+6a^2-6a+2)(a_y)\right)y+\left((-a^2+2a-1)(a_{zz})+(a-1)(a_z)^2+(a-1)(a_{yy})-(a_y)^2+(2a^2-3a+1)(a_{xx})-(a_x)^2\right)x^2+(-2a^3+8a^2-10a+4)(a_x)x-2a^4+6a^3-6a^2+2a)z^2+(\left(\left((2-2a)(a_x)(a_z)+(2a^2-4a+2)(a_{xz})\right)x+(-2a^3+6a^2-6a+2)(a_z)\right)y^2+\left(2a(a_y)(a_z)+(2a-2a^2)(a_{yz})\right)x^2 y+(2(a_x)(a_z)+(2-2a)(a_{xz}))x^3+(2-2a)(a_z)x^2)z+(a-1)(a_x)^2 y^4+\left(\left((2-2a)(a_x)(a_y)+(2a^2-4a+2)(a_{xy})\right)x+(-2a^3+6a^2-6a+2)(a_y)\right)y^3+(\left((a-1)(a_{zz})-(a_z)^2+(-a^2+2a-1)(a_{yy})+(a-1)(a_y)^2+(2a^2-3a+1)(a_{xx})-(a_x)^2\right)x^2+(-2a^3+8a^2-10a+4)(a_x)x-2a^4+6a^3-6a^2+2a)y^2+\left(2(a_x)(a_y)+(2-2a)(a_{xy})\right)x^3+(2-2a)(a_y)x^2\right)y+\left((a-1)(a_{zz})-(a_z)^2+(a-1)(a_{yy})-(a_y)^2+(a^2-2a+1)(a_{xx})\right)x^4+(2a^2-6a+4)(a_x)x^3)/((2a^3-6a^2+6a-2)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+(2a^3-6a^2+6a-2)y^4+(4a^3-12a^2+12a-4)x^2 y^2+(2a^3-6a^2+6a-2)x^4)$$

```
(%i106) ric1:ratsimp(subst(vx,diff(a,x),rics))$

(%i107) ric2:ratsimp(subst(vy,diff(a,y),ric1))$

(%i108) ric3:ratsimp(subst(vz,diff(a,z),ric2))$

(%i109) ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$

(%i110) ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$

(%i111) ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$

(%i112) ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$

(%i113) ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$

(%i114) ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

```
(%i115) rica:ratsimp(subst(v0,a,ric9));
```

$$
\begin{aligned}
(\%o115)\ & -((v0-1)\,vx^2\,z^4+\\
&(((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz)\,z^3+((2\,v0-2)\\
&vx^2\,y^2+(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy)\,y+\\
&((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(v0-1)\,vyy-vy^2+(2\,v0^2-3\,v0+1)\,vxx-vx^2)\,x^2+\\
&(-2\,v0^3+8\,v0^2-10\,v0+4)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,z^2+(\\
&(((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz)\,y^2+\\
&(2\,v0\,vy\,vz+(2\,v0-2\,v0^2)\,vyz)\,x^2\,y+(2\,vx\,vz+(2-2\,v0)\,vxz)\,x^3+(2-2\,v0)\,vz\,x^2)\,z+\\
&(v0-1)\,vx^2\,y^4+\\
&(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy)\,y^3+(\\
&((v0-1)\,vzz-vz^2+(-v0^2+2\,v0-1)\,vyy+(v0-1)\,vy^2+(2\,v0^2-3\,v0+1)\,vxx-vx^2)\,x^2+\\
&(-2\,v0^3+8\,v0^2-10\,v0+4)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,y^2+\\
&((2\,vx\,vy+(2-2\,v0)\,vxy)\,x^3+(2-2\,v0)\,vy\,x^2)\,y+\\
&((v0-1)\,vzz-vz^2+(v0-1)\,vyy-vy^2+(v0^2-2\,v0+1)\,vxx)\,x^4+(2\,v0^2-6\,v0+4)\,vx\,x^3)/\\
&((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+\\
&((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+\\
&(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\\
&x^4)
\end{aligned}
$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i116) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i117) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i118) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i119) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i120) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i121) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i122) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i123) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i124) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i125) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i126) ratsimp(rican);
(%o126) 0
```

Ripeto tutta la procedura con ric[3,3]

```
(%i127) rics:ratsimp(ric[3,3]);
```

(%o127)
$$-((a-1)(a_y)^2 z^4+\left(\left((2-2a)(a_y)(a_z)+(2a^2-4a+2)(a_{yz})\right)y+(-2a^3+6a^2-6a+2)(a_z)\right)z^3+($$
$$\left((-a^2+2a-1)(a_{zz})+(a-1)(a_z)^2+(2a^2-3a+1)(a_{yy})-(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)y^2+$$
$$\left(\left((2-2a)(a_x)(a_y)+(2a^2-4a+2)(a_{xy})\right)x+(-2a^3+8a^2-10a+4)(a_y)\right)y+(2a-2)$$
$$(a_y)^2 x^2+(-2a^3+6a^2-6a+2)(a_x)x-2a^4+6a^3-6a^2+2a)z^2+($$
$$(2(a_y)(a_z)+(2-2a)(a_{yz}))y^3+\left(\left(2a(a_x)(a_z)+(2a-2a^2)(a_{xz})\right)x+(2-2a)(a_z)\right)y^2+$$
$$\left((2-2a)(a_y)(a_z)+(2a^2-4a+2)(a_{yz})\right)x^2 y+(-2a^3+6a^2-6a+2)(a_z)x^2)z+$$
$$\left((a-1)(a_{zz})-(a_z)^2+(a^2-2a+1)(a_{yy})+(a-1)(a_{xx})-(a_x)^2\right)y^4+$$
$$\left((2(a_x)(a_y)+(2-2a)(a_{xy}))x+(2a^2-6a+4)(a_y)\right)y^3+($$
$$\left((a-1)(a_{zz})-(a_z)^2+(2a^2-3a+1)(a_{yy})-(a_y)^2+(-a^2+2a-1)(a_{xx})+(a-1)(a_x)^2\right)x^2+$$
$$(2-2a)(a_x)x)y^2+$$
$$\left(\left((2-2a)(a_x)(a_y)+(2a^2-4a+2)(a_{xy})\right)x^3+(-2a^3+8a^2-10a+4)(a_y)x^2\right)y+(a-1)$$
$$(a_y)^2 x^4+(-2a^3+6a^2-6a+2)(a_x)x^3+(-2a^4+6a^3-6a^2+2a)x^2)/($$
$$(2a^3-6a^2+6a-2)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+$$
$$(2a^3-6a^2+6a-2)y^4+(4a^3-12a^2+12a-4)x^2 y^2+(2a^3-6a^2+6a-2)x^4)$$

```
(%i128) ric1:ratsimp(subst(vx,diff(a,x),rics))$

(%i129) ric2:ratsimp(subst(vy,diff(a,y),ric1))$

(%i130) ric3:ratsimp(subst(vz,diff(a,z),ric2))$

(%i131) ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$

(%i132) ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$

(%i133) ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$

(%i134) ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$

(%i135) ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$

(%i136) ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

```
(%i137) rica:ratsimp(subst(v0,a,ric9));
```

$$
\begin{aligned}
(\%o137)\ & -((v0-1)\,vy^2\,z^4+\\
&(((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,y+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz)\,z^3+(\\
&((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(2\,v0^2-3\,v0+1)\,vyy-vy^2+(v0-1)\,vxx-vx^2)\,y^2+\\
&(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vy)\,y+(2\,v0-2)\\
&vy^2\,x^2+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,z^2+(\\
&(2\,vy\,vz+(2-2\,v0)\,vyz)\,y^3+((2\,v0\,vx\,vz+(2\,v0-2\,v0^2)\,vxz)\,x+(2-2\,v0)\,vz)\,y^2+\\
&((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,x^2\,y+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vz\,x^2)\,z+\\
&((v0-1)\,vzz-vz^2+(v0^2-2\,v0+1)\,vyy+(v0-1)\,vxx-vx^2)\,y^4+\\
&((2\,vx\,vy+(2-2\,v0)\,vxy)\,x+(2\,v0^2-6\,v0+4)\,vy)\,y^3+(\\
&((v0-1)\,vzz-vz^2+(2\,v0^2-3\,v0+1)\,vyy-vy^2+(-v0^2+2\,v0-1)\,vxx+(v0-1)\,vx^2)\,x^2+\\
&(2-2\,v0)\,vx\,x)\,y^2+\\
&(((2-2\,v0)\,vx\,vy+(2\,v0^2-4\,v0+2)\,vxy)\,x^3+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vy\,x^2)\,y+\\
&(v0-1)\,vy^2\,x^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x^3+(-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,x^2)/(\\
&(2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+\\
&((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+\\
&(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\\
&x^4)
\end{aligned}
$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i138) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i139) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i140) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i141) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i142) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i143) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i144) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i145) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i146) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i147) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i148) ratsimp(rican);
(%o148) 0
```

Ripeto tutta la procedura con ric[4,4]

```
(%i149) rics:ratsimp(ric[4,4]);
```

$$(\%o149)\ -((\left((a^2-2\,a+1)(a_{zz})+(a-1)(a_{yy})-(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)z^4+\left((2\,(a_y)(a_z)+(2-2\,a)(a_{yz}))\,y+(2\,(a_x)(a_z)+(2-2\,a)(a_{xz}))\,x+(2\,a^2-6\,a+4)(a_z)\right)z^3+(\left((2\,a^2-3\,a+1)(a_{zz})-(a_z)^2+(-a^2+2\,a-1)(a_{yy})+(a-1)(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)y^2+\left(\left(2\,a\,(a_x)(a_y)+(2\,a-2\,a^2)(a_{xy})\right)x+(2-2\,a)(a_y)\right)y+\left((2\,a^2-3\,a+1)(a_{zz})-(a_z)^2+(a-1)(a_{yy})-(a_y)^2+(-a^2+2\,a-1)(a_{xx})+(a-1)(a_x)^2\right)x^2+(2-2\,a)(a_x)\,x)\,z^2+(\left((2-2\,a)(a_y)(a_z)+(2\,a^2-4\,a+2)(a_{yz})\right)y^3+\left(\left((2-2\,a)(a_x)(a_z)+(2\,a^2-4\,a+2)(a_{xz})\right)x+(-2\,a^3+8\,a^2-10\,a+4)(a_z)\right)y^2+\left((2-2\,a)(a_y)(a_z)+(2\,a^2-4\,a+2)(a_{yz})\right)x^2\,y+\left((2-2\,a)(a_x)(a_z)+(2\,a^2-4\,a+2)(a_{xz})\right)x^3+(-2\,a^3+8\,a^2-10\,a+4)(a_z)\,x^2)\,z+(a-1)(a_z)^2\,y^4+(-2\,a^3+6\,a^2-6\,a+2)(a_y)\,y^3+\left((2\,a-2)(a_z)^2\,x^2+(-2\,a^3+6\,a^2-6\,a+2)(a_x)\,x-2\,a^4+6\,a^3-6\,a^2+2\,a\right)y^2+(-2\,a^3+6\,a^2-6\,a+2)(a_y)\,x^2\,y+(a-1)(a_z)^2\,x^4+(-2\,a^3+6\,a^2-6\,a+2)(a_x)\,x^3+(-2\,a^4+6\,a^3-6\,a^2+2\,a)\,x^2)/((2\,a^3-6\,a^2+6\,a-2)\,z^4+((4\,a^3-12\,a^2+12\,a-4)\,y^2+(4\,a^3-12\,a^2+12\,a-4)\,x^2)\,z^2+(2\,a^3-6\,a^2+6\,a-2)\,y^4+(4\,a^3-12\,a^2+12\,a-4)\,x^2\,y^2+(2\,a^3-6\,a^2+6\,a-2)\,x^4)$$

```
(%i150) ric1:ratsimp(subst(vx,diff(a,x),rics))$

(%i151) ric2:ratsimp(subst(vy,diff(a,y),ric1))$

(%i152) ric3:ratsimp(subst(vz,diff(a,z),ric2))$

(%i153) ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$

(%i154) ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$

(%i155) ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$

(%i156) ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$

(%i157) ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$

(%i158) ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

```
(%i159) rica:ratsimp(subst(v0,a,ric9));
```

$$(\%o159)\ -(((v0^2-2\,v0+1)\,vzz+(v0-1)\,vyy-vy^2+(v0-1)\,vxx-vx^2)\,z^4+((2\,vy\,vz+(2-2\,v0)\,vyz)\,y+(2\,vx\,vz+(2-2\,v0)\,vxz)\,x+(2\,v0^2-6\,v0+4)\,vz)\,z^3+(((2\,v0^2-3\,v0+1)\,vzz-vz^2+(-v0^2+2\,v0-1)\,vyy+(v0-1)\,vy^2+(v0-1)\,vxx-vx^2)\,y^2+((2\,v0\,vx\,vy+(2\,v0-2\,v0^2)\,vxy)\,x+(2-2\,v0)\,vy)\,y+((2\,v0^2-3\,v0+1)\,vzz-vz^2+(v0-1)\,vyy-vy^2+(-v0^2+2\,v0-1)\,vxx+(v0-1)\,vx^2)\,x^2+(2-2\,v0)\,vx\,x)\,z^2+(((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,y^3+(((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vz)\,y^2+((2-2\,v0)\,vy\,vz+(2\,v0^2-4\,v0+2)\,vyz)\,x^2\,y+((2-2\,v0)\,vx\,vz+(2\,v0^2-4\,v0+2)\,vxz)\,x^3+(-2\,v0^3+8\,v0^2-10\,v0+4)\,vz\,x^2)\,z+(v0-1)\,vz^2\,y^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy\,y^3+((2\,v0-2)\,vz^2\,x^2+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,y^2+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vy\,x^2\,y+(v0-1)\,vz^2\,x^4+(-2\,v0^3+6\,v0^2-6\,v0+2)\,vx\,x^3+(-2\,v0^4+6\,v0^3-6\,v0^2+2\,v0)\,x^2)\,/\,((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i160) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i161) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i162) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i163) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i164) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i165) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i166) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i167) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i168) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i169) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i170) ratsimp(rican);
(%o170) 0
```

Ora verifico anche i termini NON DIAGONALI che possono essere diversi da zero se la funzione non possiede le opportune caratteristiche.

(%i171)
```
rics:ratsimp(ric[2,3]);
```

(%o171)
$$-((a-1)(a_x)(a_y)z^4+\left(\left((1-a)(a_x)(a_z)+(a^2-2a+1)(a_{xz})\right)y+\left((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz})\right)x\right)z^3+(\left((a-1)(a_x)(a_y)+(a^2-2a+1)(a_{xy})\right)y^2+(\left((-a^2+2a-1)(a_{zz})+(a-1)(a_z)^2+(a^2-a)(a_{yy})-a(a_y)^2+(a^2-a)(a_{xx})-a(a_x)^2\right)x+(a^2-2a+1)(a_x))y+\left((a-1)(a_x)(a_y)+(a^2-2a+1)(a_{xy})\right)x^2+(a^2-2a+1)(a_y)x)z^2+(\left((1-a)(a_x)(a_z)+(a^2-2a+1)(a_{xz})\right)y^3+\left((a+1)(a_y)(a_z)+(1-a^2)(a_{yz})\right)xy^2+\left(\left((a+1)(a_x)(a_z)+(1-a^2)(a_{xz})\right)x^2+(2a^3-6a^2+4a)(a_z)x\right)y+\left((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz})\right)x^3)z+(a^2-2a+1)(a_{xy})y^4+\left(\left((a-1)(a_{zz})-(a_z)^2+(a^2-a)(a_{xx})-a(a_x)^2\right)x+(a^2-2a+1)(a_x)\right)y^3+\left((2a(a_x)(a_y)+(2-2a)(a_{xy}))x^2+(2a^3-5a^2+2a+1)(a_y)x\right)y^2+(\left((a-1)(a_{zz})-(a_z)^2+(a^2-a)(a_{yy})-a(a_y)^2\right)x^3+(2a^3-5a^2+2a+1)(a_x)x^2+(2a^4-6a^3+6a^2-2a)x)y+(a^2-2a+1)(a_{xy})x^4+(a^2-2a+1)(a_y)x^3)/(\left(2a^3-6a^2+6a-2\right)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+\left(2a^3-6a^2+6a-2\right)y^4+(4a^3-12a^2+12a-4)x^2y^2+(2a^3-6a^2+6a-2)x^4)$$

(%i172)
```
ric1:ratsimp(subst(vx,diff(a,x),rics))$
```

(%i173)
```
ric2:ratsimp(subst(vy,diff(a,y),ric1))$
```

(%i174)
```
ric3:ratsimp(subst(vz,diff(a,z),ric2))$
```

(%i175)
```
ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$
```

(%i176)
```
ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$
```

(%i177)
```
ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$
```

(%i178)
```
ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$
```

(%i179)
```
ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$
```

(%i180)
```
ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

```
(%i181) rica:ratsimp(subst(v0,a,ric9));
```

(%o181)

$$-((v0-1)\,vx\,vy\,z^4+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,y+((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x)\,z^3+((v0-1)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y^2+((-v0^2+2\,v0-1)\,vzz+(v0-1)\,vz^2+(v0^2-v0)\,vyy-v0\,vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,y+((v0-1)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x^2+(v0^2-2\,v0+1)\,vy\,x)\,z^2+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,y^3+((v0+1)\,vy\,vz+(1-v0^2)\,vyz)\,x\,y^2+(((v0+1)\,vx\,vz+(1-v0^2)\,vxz)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vz\,x)\,y+((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^3)\,z+(v0^2-2\,v0+1)\,vxy\,y^4+(((v0-1)\,vzz-vz^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,y^3+((2\,v0\,vx\,vy+(2-2\,v0)\,vxy)\,x^2+(2\,v0^3-5\,v0^2+2\,v0+1)\,vy\,x)\,y^2+((v0-1)\,vzz-vz^2+(v0^2-v0)\,vyy-v0\,vy^2)\,x^3+(2\,v0^3-5\,v0^2+2\,v0+1)\,vx\,x^2+(2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,x)\,y+(v0^2-2\,v0+1)\,vxy\,x^4+(v0^2-2\,v0+1)\,vy\,x^3)/((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i182) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i183) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i184) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i185) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i186) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i187) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i188) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i189) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i190) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i191) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i192) ratsimp(rican);
(%o192) 0
```

Verifico anche il termine NON DIAGONALE r[2,4] che puo' essere diverso da zero se la funzione non possiede le opportune caratteristiche.

```
(%i193) rics:ratsimp(ric[2,4]);
```

$$(\%o193)\ -((a^2-2a+1)(a_{xz})z^4+(((1-a)(a_x)(a_y)+(a^2-2a+1)(a_{xy}))y+((a-1)(a_{yy})-(a_y)^2+(a^2-a)(a_{xx})-a(a_x)^2)x+(a^2-2a+1)(a_x))z^3+(((a-1)(a_x)(a_z)+(a^2-2a+1)(a_{xz}))y^2+(((a+1)(a_y)(a_z)+(1-a^2)(a_{yz}))xy+(2a(a_x)(a_z)+(2-2a)(a_{xz}))x^2+(2a^3-5a^2+2a+1)(a_z)x)z^2+(((1-a)(a_x)(a_y)+(a^2-2a+1)(a_{xy}))y^3+(((a^2-a)(a_{zz})-a(a_z)^2+(-a^2+2a-1)(a_{yy})+(a-1)(a_y)^2+(a^2-a)(a_{xx})-a(a_x)^2)x+(a^2-2a+1)(a_x))y^2+(((a+1)(a_x)(a_y)+(1-a^2)(a_{xy}))x^2+(2a^3-6a^2+4a)(a_y)x)y+((a^2-a)(a_{zz})-a(a_z)^2+(a-1)(a_{yy})-(a_y)^2)x^3+(2a^3-5a^2+2a+1)(a_x)x^2+(2a^4-6a^3+6a^2-2a)x)z+(a-1)(a_x)(a_z)y^4+(((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz}))xy^3+(((a-1)(a_x)(a_z)+(a^2-2a+1)(a_{xz}))x^2+(a^2-2a+1)(a_z)x)y^2+((1-a)(a_y)(a_z)+(a^2-2a+1)(a_{yz}))x^3y+(a^2-2a+1)(a_{xz})x^4+(a^2-2a+1)(a_z)x^3)/((2a^3-6a^2+6a-2)z^4+((4a^3-12a^2+12a-4)y^2+(4a^3-12a^2+12a-4)x^2)z^2+(2a^3-6a^2+6a-2)y^4+(4a^3-12a^2+12a-4)x^2y^2+(2a^3-6a^2+6a-2)x^4)$$

```
(%i194) ric1:ratsimp(subst(vx,diff(a,x),rics))$

(%i195) ric2:ratsimp(subst(vy,diff(a,y),ric1))$

(%i196) ric3:ratsimp(subst(vz,diff(a,z),ric2))$

(%i197) ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$

(%i198) ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$

(%i199) ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$

(%i200) ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$

(%i201) ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$

(%i202) ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

```
(%i203) rica:ratsimp(subst(v0,a,ric9));
```

(%o203)
$$-((v0^2-2\,v0+1)\,vxz\,z^4+(((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y+((v0-1)\,vyy-vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,z^3+(((v0-1)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,y^2+((v0+1)\,vy\,vz+(1-v0^2)\,vyz)\,x\,y+(2\,v0\,vx\,vz+(2-2\,v0)\,vxz)\,x^2+(2\,v0^3-5\,v0^2+2\,v0+1)\,vz\,x)\,z^2+(((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,y^3+((v0^2-v0)\,vzz-v0\,vz^2+(-v0^2+2\,v0-1)\,vyy+(v0-1)\,vy^2+(v0^2-v0)\,vxx-v0\,vx^2)\,x+(v0^2-2\,v0+1)\,vx)\,y^2+(((v0+1)\,vx\,vy+(1-v0^2)\,vxy)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vy\,x)\,y+((v0^2-v0)\,vzz-v0\,vz^2+(v0-1)\,vyy-vy^2)\,x^3+(2\,v0^3-5\,v0^2+2\,v0+1)\,vx\,x^2+(2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,x)\,z+(v0-1)\,vx\,vz\,y^4+((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x\,y^3+(((v0-1)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x^2+(v0^2-2\,v0+1)\,vz\,x)\,y^2+((1-v0)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^3\,y+(v0^2-2\,v0+1)\,vxz\,x^4+(v0^2-2\,v0+1)\,vz\,x^3)/((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

```
(%i204) ric1n:ratsimp(subst(vxn,vx,rica))$

(%i205) ric2n:ratsimp(subst(vyn,vy,ric1n))$

(%i206) ric3n:ratsimp(subst(vzn,vz,ric2n))$

(%i207) ric4n:ratsimp(subst(vxxn,vxx,ric3n))$

(%i208) ric5n:ratsimp(subst(vyyn,vyy,ric4n))$

(%i209) ric6n:ratsimp(subst(vzzn,vzz,ric5n))$

(%i210) ric7n:ratsimp(subst(vxyn,vxy,ric6n))$

(%i211) ric8n:ratsimp(subst(vxzn,vxz,ric7n))$

(%i212) ric9n:ratsimp(subst(vyzn,vyz,ric8n))$

(%i213) rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i214) ratsimp(rican);
(%o214) 0
```

Ora verifico anche il termine NON DIAGONALE r[3,4] che potrebbe essere dive
da zero se la funzione non possiede le opportune caratteristiche.

```
(%i215) rics:ratsimp(ric[3,4]);
```

$$(\%o215)\ -((a^2-2\,a+1)(a_{yz})\,z^4+(\left((a^2-a)(a_{yy})-a\,(a_y)^2+(a-1)(a_{xx})-(a_x)^2\right)y+$$
$$\left((1-a)(a_x)(a_y)+(a^2-2\,a+1)(a_{xy})\right)x+(a^2-2\,a+1)(a_y))\,z^3+($$
$$(2\,a\,(a_y)(a_z)+(2-2\,a)(a_{yz}))\,y^2+$$
$$\left(\left((a+1)(a_x)(a_z)+(1-a^2)(a_{xz})\right)x+(2\,a^3-5\,a^2+2\,a+1)(a_z)\right)y+$$
$$\left((a-1)(a_y)(a_z)+(a^2-2\,a+1)(a_{yz})\right)x^2)\,z^2+($$
$$\left((a^2-a)(a_{zz})-a\,(a_z)^2+(a-1)(a_{xx})-(a_x)^2\right)y^3+$$
$$\left(\left((a+1)(a_x)(a_y)+(1-a^2)(a_{xy})\right)x+(2\,a^3-5\,a^2+2\,a+1)(a_y)\right)y^2+($$
$$\left((a^2-a)(a_{zz})-a\,(a_z)^2+(a^2-a)(a_{yy})-a\,(a_y)^2+(-a^2+2\,a-1)(a_{xx})+(a-1)(a_x)^2\right)x^2+$$
$$(2\,a^3-6\,a^2+4\,a)(a_x)\,x+2\,a^4-6\,a^3+6\,a^2-2\,a)\,y+\left((1-a)(a_x)(a_y)+(a^2-2\,a+1)(a_{xy})\right)$$
$$x^3+(a^2-2\,a+1)(a_y)\,x^2)\,z+(a^2-2\,a+1)(a_{yz})\,y^4+$$
$$\left(\left((1-a)(a_x)(a_z)+(a^2-2\,a+1)(a_{xz})\right)x+(a^2-2\,a+1)(a_z)\right)y^3+$$
$$\left((a-1)(a_y)(a_z)+(a^2-2\,a+1)(a_{yz})\right)x^2\,y^2+$$
$$\left(\left((1-a)(a_x)(a_z)+(a^2-2\,a+1)(a_{xz})\right)x^3+(a^2-2\,a+1)(a_z)\,x^2\right)y+(a-1)(a_y)(a_z)\,x^4)/($$
$$(2\,a^3-6\,a^2+6\,a-2)\,z^4+((4\,a^3-12\,a^2+12\,a-4)\,y^2+(4\,a^3-12\,a^2+12\,a-4)\,x^2)\,z^2+$$
$$(2\,a^3-6\,a^2+6\,a-2)\,y^4+(4\,a^3-12\,a^2+12\,a-4)\,x^2\,y^2+(2\,a^3-6\,a^2+6\,a-2)\,x^4)$$

```
(%i216) ric1:ratsimp(subst(vx,diff(a,x),rics))$

(%i217) ric2:ratsimp(subst(vy,diff(a,y),ric1))$

(%i218) ric3:ratsimp(subst(vz,diff(a,z),ric2))$

(%i219) ric4:ratsimp(subst(vxx,diff(a,x,2),ric3))$

(%i220) ric5:ratsimp(subst(vyy,diff(a,y,2),ric4))$

(%i221) ric6:ratsimp(subst(vzz,diff(a,z,2),ric5))$

(%i222) ric7:ratsimp(subst(vxy,diff(diff(a,x),y),ric6))$

(%i223) ric8:ratsimp(subst(vxz,diff(diff(a,x),z),ric7))$

(%i224) ric9:ratsimp(subst(vyz,diff(diff(a,y),z),ric8))$
```

(%i225)
```
rica:ratsimp(subst(v0,a,ric9));
```

(%o225)
$$-((v0^2-2\,v0+1)\,vyz\,z^4+(((v0^2-v0)\,vyy-v0\,vy^2+(v0-1)\,vxx-vx^2)\,y+((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x+(v0^2-2\,v0+1)\,vy)\,z^3+((2\,v0\,vy\,vz+(2-2\,v0)\,vyz)\,y^2+(((v0+1)\,vx\,vz+(1-v0^2)\,vxz)\,x+(2\,v0^3-5\,v0^2+2\,v0+1)\,vz)\,y+((v0-1)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^2)\,z^2+((((v0^2-v0)\,vzz-v0\,vz^2+(v0-1)\,vxx-vx^2)\,y^3+(((v0+1)\,vx\,vy+(1-v0^2)\,vxy)\,x+(2\,v0^3-5\,v0^2+2\,v0+1)\,vy)\,y^2+((((v0^2-v0)\,vzz-v0\,vz^2+(v0^2-v0)\,vyy-v0\,vy^2+(-v0^2+2\,v0-1)\,vxx+(v0-1)\,vx^2)\,x^2+(2\,v0^3-6\,v0^2+4\,v0)\,vx\,x+2\,v0^4-6\,v0^3+6\,v0^2-2\,v0)\,y+((1-v0)\,vx\,vy+(v0^2-2\,v0+1)\,vxy)\,x^3+(v0^2-2\,v0+1)\,vy\,x^2)\,z+(v0^2-2\,v0+1)\,vyz\,y^4+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x+(v0^2-2\,v0+1)\,vz)\,y^3+((v0-1)\,vy\,vz+(v0^2-2\,v0+1)\,vyz)\,x^2\,y^2+(((1-v0)\,vx\,vz+(v0^2-2\,v0+1)\,vxz)\,x^3+(v0^2-2\,v0+1)\,vz\,x^2)\,y+(v0-1)\,vy\,vz\,x^4)/((2\,v0^3-6\,v0^2+6\,v0-2)\,z^4+((4\,v0^3-12\,v0^2+12\,v0-4)\,y^2+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2)\,z^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,y^4+(4\,v0^3-12\,v0^2+12\,v0-4)\,x^2\,y^2+(2\,v0^3-6\,v0^2+6\,v0-2)\,x^4)$$

Ora sostituisco ai simboli le vere funzioni dedotte a partire da v0n.

(%i226)
```
ric1n:ratsimp(subst(vxn,vx,rica))$
```

(%i227)
```
ric2n:ratsimp(subst(vyn,vy,ric1n))$
```

(%i228)
```
ric3n:ratsimp(subst(vzn,vz,ric2n))$
```

(%i229)
```
ric4n:ratsimp(subst(vxxn,vxx,ric3n))$
```

(%i230)
```
ric5n:ratsimp(subst(vyyn,vyy,ric4n))$
```

(%i231)
```
ric6n:ratsimp(subst(vzzn,vzz,ric5n))$
```

(%i232)
```
ric7n:ratsimp(subst(vxyn,vxy,ric6n))$
```

(%i233)
```
ric8n:ratsimp(subst(vxzn,vxz,ric7n))$
```

(%i234)
```
ric9n:ratsimp(subst(vyzn,vyz,ric8n))$
```

(%i235)
```
rican:ratsimp(subst(v0n,v0,ric9n))$
```

Ecco il test cruciale: sostituendo e sostituendo alla fine viene zero o cosa vien fuori ?

```
(%i236) ratsimp(rican);
(%o236) 0
```

Per finire ristampo il tensore metrico covariante che ho usato per trovare la metrica del buco nero neutro in coordinate cartesiane.

```
(%i237) lg;
(%o237)
```

$$\begin{bmatrix} 1-a & 0 & 0 & 0 \\ 0 & -\frac{(a-1)\,z^2+(a-1)\,y^2-x^2}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,x\,y}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,x\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} \\ 0 & \frac{a\,x\,y}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & -\frac{(a-1)\,z^2-y^2+(a-1)\,x^2}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,y\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} \\ 0 & \frac{a\,x\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{a\,y\,z}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} & \frac{z^2+(1-a)\,y^2+(1-a)\,x^2}{(a-1)\,z^2+(a-1)\,y^2+(a-1)\,x^2} \end{bmatrix}$$

E per riepilogo ecco anche il tensore metrico in forma controvariante:

```
(%i238) ug;
```

$$\begin{bmatrix} -\frac{1}{a-1} & 0 & 0 & 0 \\ 0 & -\frac{z^2+y^2+(1-a)\,x^2}{z^2+y^2+x^2} & \frac{a\,x\,y}{z^2+y^2+x^2} & \frac{a\,x\,z}{z^2+y^2+x^2} \\ 0 & \frac{a\,x\,y}{z^2+y^2+x^2} & -\frac{z^2+(1-a)\,y^2+x^2}{z^2+y^2+x^2} & \frac{a\,y\,z}{z^2+y^2+x^2} \\ 0 & \frac{a\,x\,z}{z^2+y^2+x^2} & \frac{a\,y\,z}{z^2+y^2+x^2} & \frac{(a-1)\,z^2-y^2-x^2}{z^2+y^2+x^2} \end{bmatrix}$$

```
(%o238)
```