Prove di calcolo tensoriale

Provo a realizzare un documento wxm per il calcolo di tutte le grandezze derivate da un assegnato tensore metrico.
In questo caso uso il tensore metrico di Reissner e Nordstrom.

```
(%i1) (if atom(lg) then load(ctensor));
(%o1)
C:/Programmi/Maxima-5.20.1/share/maxima/5.20.1/share/tensor/ctensor.mac
```

```
(%i2) init_ctensor();
(%o2) done
```

```
(%i3) ct_coords: [t,r,theta,phi];
(%o3) [t,r,theta,phi]
```

```
(%i4) lg:matrix(
[1-2*m/r+(q/r)^2,0,0,0],
[0,-1/(1 -2*m/r+(q/r)^2),0,0],
[0,0,-r^2,0],
[0,0,0,-(r*sin(theta))^2]);
```

(%o4)
$$\begin{bmatrix} -\frac{2\,m}{r}+\frac{q^2}{r^2}+1 & 0 & 0 & 0 \\ 0 & -\frac{1}{-\frac{2\,m}{r}+\frac{q^2}{r^2}+1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\sin(\mathrm{theta})^2 \end{bmatrix}$$

```
(%i5) lg;
```

(%o5)
$$\begin{bmatrix} -\frac{2\,m}{r}+\frac{q^2}{r^2}+1 & 0 & 0 & 0 \\ 0 & -\frac{1}{-\frac{2\,m}{r}+\frac{q^2}{r^2}+1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\sin(\mathrm{theta})^2 \end{bmatrix}$$

```
(%i6) cmetric();
(%o6) done
```

(%i7) ug;

(%o7)
$$\begin{bmatrix} \frac{1}{-\frac{2\,m}{r}+\frac{q^2}{r^2}+1} & 0 & 0 & 0 \\ 0 & \frac{2\,m}{r}-\frac{q^2}{r^2}-1 & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\,\sin(\mathrm{theta})^2} \end{bmatrix}$$

(%i8) christof(all);

(%t8) $lcs_{1,1,2}=\frac{\frac{2\,q^2}{r^3}-\frac{2\,m}{r^2}}{2}$

(%t9) $lcs_{1,2,1}=\frac{\frac{2\,m}{r^2}-\frac{2\,q^2}{r^3}}{2}$

(%t10) $lcs_{2,2,2}=\frac{\frac{2\,m}{r^2}-\frac{2\,q^2}{r^3}}{2\left(-\frac{2\,m}{r}+\frac{q^2}{r^2}+1\right)^2}$

(%t11) $lcs_{2,3,3}=-r$

(%t12) $lcs_{2,4,4}=-r\,\sin(\mathrm{theta})^2$

(%t13) $lcs_{3,3,2}=r$

(%t14) $lcs_{3,4,4}=-r^2\cos(\mathrm{theta})\sin(\mathrm{theta})$

(%t15) $lcs_{4,4,2}=r\,\sin(\mathrm{theta})^2$

(%t16) $lcs_{4,4,3}=r^2\cos(\mathrm{theta})\sin(\mathrm{theta})$

(%t17) $mcs_{1,1,2}=\frac{m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4}{r^5}$

(%t18) $mcs_{1,2,1}=\frac{m\,r-q^2}{r^3-2\,m\,r^2+q^2\,r}$

(%t19) $mcs_{2,2,2}=-\frac{m\,r-q^2}{r^3-2\,m\,r^2+q^2\,r}$

(%t20) $mcs_{2,3,3}=\frac{1}{r}$

(%t21) $mcs_{2,4,4}=\frac{1}{r}$

(%t22) $mcs_{3,3,2}=-\frac{r^2-2\,m\,r+q^2}{r}$

(%t23) $mcs_{3,4,4}=\frac{\cos(\mathrm{theta})}{\sin(\mathrm{theta})}$

(%t24) $mcs_{4,4,2}=-\frac{(r^2-2\,m\,r+q^2)\,\sin(\mathrm{theta})^2}{r}$

(%t25) $mcs_{4,4,3}=-\cos(\mathrm{theta})\sin(\mathrm{theta})$

(%o25) *done*

(%i26) riemann(true);

(%t26) $riem_{1,2,1,2}=\frac{2\,(m\,r-q^2)\,(m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4)}{r^5\,(r^3-2\,m\,r^2+q^2\,r)}+\frac{5\,(m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4)}{r^6}-\frac{3\,m\,r^2+2\,(-q^2-2\,m^2)\,r+3\,m\,q^2}{r^5}$

(%t27) $riem_{1,3,1,3}=-\frac{m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4}{r^6}$

(%t28) $riem_{1,4,1,4}=-\frac{m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4}{r^6}$

(%t29) $riem_{2,2,1,1}=\frac{2\,m\,r-3\,q^2}{r^4-2\,m\,r^3+q^2\,r^2}$

(%t30) $riem_{2,3,2,3}=\frac{m\,r-q^2}{r\,(r^3-2\,m\,r^2+q^2\,r)}$

(%t31) $riem_{2,4,2,4}=\frac{m\,r-q^2}{r\,(r^3-2\,m\,r^2+q^2\,r)}$

(%t32) $riem_{3,3,1,1}=-\frac{m\,r-q^2}{r^2}$

(%t33) $riem_{3,3,2,2}=-\frac{m\,r-q^2}{r^2}$

(%t34) $riem_{3,4,3,4}=\frac{r^2-2\,m\,r+q^2}{r^2}-1$

(%t35) $riem_{4,4,1,1}=-\frac{(m\,r-q^2)\,\sin(\theta)^2}{r^2}$

(%t36) $riem_{4,4,2,2}=-\frac{(m\,r-q^2)\,\sin(\theta)^2}{r^2}$

(%t37) $riem_{4,4,3,3}=\frac{(2\,m\,r-q^2)\,\sin(\theta)^2}{r^2}$

(%o37) done

(%i38) ricci(true);

(%t38) $ric_{1,1} = -\frac{2\,(m\,r-q^2)\,(m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4)}{r^5\,(r^3-2\,m\,r^2+q^2\,r)} - \frac{3\,(m\,r^3+(-q^2-2\,m^2)\,r^2+3\,m\,q^2\,r-q^4)}{r^6} + \frac{3\,m\,r^2+2\,(-q^2-2\,m^2)\,r+3\,m\,q^2}{r^5}$

(%t39) $ric_{2,2} = -\frac{2\,(m\,r-q^2)}{r\,(r^3-2\,m\,r^2+q^2\,r)} - \frac{m}{r^3-2\,m\,r^2+q^2\,r} + \frac{(m\,r-q^2)\,(3\,r^2-4\,m\,r+q^2)}{(r^3-2\,m\,r^2+q^2\,r)^2} - \frac{2\,(m\,r-q^2)^2}{(r^3-2\,m\,r^2+q^2\,r)^2}$

(%t40) $ric_{3,3} = \frac{r^2-2\,m\,r+q^2}{r^2} - \frac{2\,r-2\,m}{r} + 1$

(%t41) $ric_{4,4} = \frac{(r^2-2\,m\,r+q^2)\,\sin(\theta)^2}{r^2} - \frac{(2\,r-2\,m)\,\sin(\theta)^2}{r} + \sin(\theta)^2$

(%o41) *done*

(%i42) ratsimp(listarray(ric));

(%o42) $\left[\frac{q^2\,r^2-2\,m\,q^2\,r+q^4}{r^6}, 0, 0, 0, 0, -\frac{q^2}{r^4-2\,m\,r^3+q^2\,r^2}, 0, 0, 0, 0, \frac{q^2}{r^2}, 0, 0, 0, 0, \frac{q^2\,\sin(\theta)^2}{r^2}\right]$

Come si vede il tensore di Ricci dipende dal valore di q ossia
se la carica è nulla ovvero q=0, allora la metrica di Reissner e
Nordstrom produce un tensore di Ricci identicamente nullo
ovvero la soluzione del buco nero neutro altrimenti detta la
metrica di Schwarzschild.

(%i43)
```
mat_ricci: ratsimp( matrix(
[ ric[1,1],ric[1,2],ric[1,3],ric[1,4]],
[ ric[2,1],ric[2,2],ric[2,3],ric[2,4]],
[ ric[3,1],ric[3,2],ric[3,3],ric[3,4]],
[ ric[4,1],ric[4,2],ric[4,3],ric[4,4]]));
```

(%o43) $\begin{bmatrix} \frac{q^2\,r^2-2\,m\,q^2\,r+q^4}{r^6} & 0 & 0 & 0 \\ 0 & -\frac{q^2}{r^4-2\,m\,r^3+q^2\,r^2} & 0 & 0 \\ 0 & 0 & \frac{q^2}{r^2} & 0 \\ 0 & 0 & 0 & \frac{q^2\,\sin(\theta)^2}{r^2} \end{bmatrix}$

...etc...