Rassegna di tensori metrici predefiniti in ctensor.

Visualizzo i tensori metrici offerti di default dal package ctensor.

Carico il package:

```
(%i1) if atom(lg) then load(ctensor);
(%o1)
C:/PROGRA~2/MAXIMA~1.0/share/maxima/5.21.0/share/tensor/ctensor.mac
```

```
(%i2) lg:["Non ricaricare il package ctensor"];
(%o2) [Non ricaricare il package ctensor]
```

```
(%i3) cframe_flag;
(%o3) false
```

In base al manuale elenco le varie opzioni disponibili:

```
(%i4) opzioni:[cartesian2d,polar,elliptic,confocalelliptic,bipolar,
      parabolic,cartesian3d,polarcylindrical,paraboloidal,
      conical,toroidal,spherical,oblatespheroidal,
      oblatespheroidalsqrt,prolatespheroidal,prolatespheroidalsqrt,
      ellipsoidal,cartesian4d,spherical4d,exteriorschwarzschild,
      interiorschwarzschild,kerr_newman];
(%o4) [cartesian2d,polar,elliptic,confocalelliptic,bipolar,parabolic,
cartesian3d,polarcylindrical,paraboloidal,conical,toroidal,spherical,
oblatespheroidal,oblatespheroidalsqrt,prolatespheroidal,
prolatespheroidalsqrt,ellipsoidal,cartesian4d,spherical4d,
exteriorschwarzschild,interiorschwarzschild,kerr_newman]
```

```
(%i5) length(opzioni);
(%o5) 22
```

Scrivo una funzione che carica in sequenza tutti i sistemi selezionabili tramite gli elementi della lista opzioni e conserva nella lista lg_vari i vari tensori metrici in forma covariante.

```
(%i6) lgvari(oa,ob):=block([j,inpiu],
          lg_vari:[["Tipo di sistema","Matrice del tensore metrico"]],
          for j:1 thru length(opzioni) do (
          ct_coordsys(opzioni[j]),
          inpiu:[[lg,j,opzioni[j]]],
          lg_vari:append(lg_vari,inpiu)
          ),
          print("Ha generato la lista lg_vari"),
          for j:oa+1 thru min(ob+1,length(lg_vari)) do
          print(lg_vari[j])
      )$
```

```
(%i7) lgvari(1,7);
```

Ha generato la lista lg_vari

$$\left[\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, 1, \textit{cartesian2d}\right]$$

$$\left[\begin{bmatrix}1 & 0\\ 0 & r^2\end{bmatrix}, 2, \textit{polar}\right]$$

$$\left[\begin{bmatrix}e^2\left(\cosh(u)^2-\cos(v)^2\right) & 0\\ 0 & e^2\left(\cosh(u)^2-\cos(v)^2\right)\end{bmatrix}, 3, \textit{elliptic}\right]$$

$$\left[\begin{bmatrix}\frac{e^2\left(u^2-v^2\right)}{u^2-1} & 0\\ 0 & \frac{e^2\left(v^2-u^2\right)}{v^2-1}\end{bmatrix}, 4, \textit{confocalelliptic}\right]$$

$$\left[\begin{bmatrix}\frac{e^2}{\left(\cosh(v)-\cos(u)\right)^2} & 0\\ 0 & \frac{e^2}{\left(\cosh(v)-\cos(u)\right)^2}\end{bmatrix}, 5, \textit{bipolar}\right]$$

$$\left[\begin{bmatrix}v^2+u^2 & 0\\ 0 & v^2+u^2\end{bmatrix}, 6, \textit{parabolic}\right]$$

$$\left[\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, 7, \textit{cartesian3d}\right]$$

(%o7) *done*

```
(%i8) lgvari(8,13);
```

Ha generato la lista lg_vari

$$\left[\begin{pmatrix}1&0&0\\0&r^2&0\\0&0&1\end{pmatrix},8,\textit{polarcylindrical}\right]$$

$$\left[\begin{pmatrix}u^2\,v^2&0&0\\0&v^2+u^2&0\\0&0&v^2+u^2\end{pmatrix},9,\textit{paraboloidal}\right]$$

$$\left[\begin{pmatrix}\frac{(v-u)\,(v+u)\,w^2}{(u-e)\,(u+e)\,(u-f)\,(u+f)}&0&0\\0&\frac{(u-v)\,(v+u)\,w^2}{(v-e)\,(v+e)\,(v-f)\,(v+f)}&0\\0&0&1\end{pmatrix},10,\textit{conical}\right]$$

$$\left[\begin{pmatrix}\frac{e^2\,\sinh(v)^2}{(\cosh(v)-\cos(u))^2}&0&0\\0&\frac{e^2}{(\cosh(v)-\cos(u))^2}&0\\0&0&\frac{e^2}{(\cosh(v)-\cos(u))^2}\end{pmatrix},11,\textit{toroidal}\right]$$

$$\left[\begin{pmatrix}1&0&0\\0&r^2&0\\0&0&r^2\,\sin(\theta)^2\end{pmatrix},12,\textit{spherical}\right]$$

$$\left[\begin{pmatrix}e^2\,(\sin(v)^2+\sinh(u)^2)&0&0\\0&e^2\,(\sin(v)^2+\sinh(u)^2)&0\\0&0&e^2\,\cosh(u)^2\,\cos(v)^2\end{pmatrix},13,\textit{oblatespheroidal}\right]$$

]

```
(%o8) done
```

```
(%i9) lgvari(14,18);
```

Ha generato la lista lg_vari

$$\left[\begin{bmatrix} \frac{e^2\,(u^2-v^2)}{u^2-1} & 0 & 0 \\ 0 & \frac{e^2\,(u^2-v^2)}{u^2-1} & 0 \\ 0 & 0 & e^2\,u^2\,v^2 \end{bmatrix},14,\mathit{oblatespheroidalsqrt}\right]$$

$$\left[\begin{bmatrix} e^2\,(\sin(v)^2+\sinh(u)^2) & 0 & 0 \\ 0 & e^2\,(\sin(v)^2+\sinh(u)^2) & 0 \\ 0 & 0 & e^2\,\sinh(u)^2\,\sin(v)^2 \end{bmatrix},15,\right.$$

$$\left.\mathit{prolatespheroidal}\right]$$

$$\left[\begin{bmatrix} \frac{e^2\,(v^2-u^2)}{1-u^2} & 0 & 0 \\ 0 & \frac{e^2\,(v^2-u^2)}{v^2-1} & 0 \\ 0 & 0 & e^2\,(1-u^2)\,(v^2-1) \end{bmatrix},16,\mathit{prolatespheroidalsqrt}\right]$$

$$\left[\begin{bmatrix} (b^2\,\sin(\phi)^2+a^2\,\cos(\phi)^2)\,\sin(\theta)^2+c^2\,\cos(\theta)^2 & (b^2\,\sin(\phi)^2+a^2\,\cos(\phi)^2-c^2)\,r\,\cos(\theta)\,\sin(\theta) & (b^2-a^2) \\ (b^2\,\sin(\phi)^2+a^2\,\cos(\phi)^2-c^2)\,r\,\cos(\theta)\,\sin(\theta) & r^2\,(c^2\,\sin(\theta)^2+(b^2\,\sin(\phi)^2+a^2\,\cos(\phi)^2)\,\cos(\theta)^2) & (b^2-a^2)\,\cos \\ (b^2-a^2)\,\cos(\phi)\,\sin(\phi)\,r\,\sin(\theta)^2 & (b^2-a^2)\,\cos(\phi)\,\sin(\phi)\,r^2\,\cos(\theta)\,\sin(\theta) & (a^2\,\sin(\phi) \end{bmatrix},17,\mathit{ellipsoidal}\right]$$

$$\left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},18,\mathit{cartesian4d}\right]$$

```
(%o9) done
```

```
(%i10) lgvari(19,25);
```

Ha generato la lista lg_vari

$$\left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 & 0 \\ 0 & 0 & 0 & \sin(\eta)^2\, r^2\sin(\theta)^2 \end{bmatrix}, 19, \textit{spherical4d}\right]$$

$$\left[\begin{bmatrix} \frac{2m-r}{r} & 0 & 0 & 0 \\ 0 & \frac{r}{r-2m} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)^2 \end{bmatrix}, 20, \textit{exteriorschwarzschild}\right]$$

$$\left[\begin{bmatrix} -\frac{t}{2m-t} & 0 & 0 & 0 \\ 0 & \frac{2m-t}{t} & 0 & 0 \\ 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & t^2\sin(u)^2 \end{bmatrix}, 21, \textit{interiorschwarzschild}\right]$$

$$\left[\begin{bmatrix} \frac{a^2\sin(\theta)^2-r^2+2mr-e^2-a^2}{a^2\cos(\theta)^2+r^2} & 0 & 0 & \frac{a(e^2-2mr)\sin(\theta)^2}{a^2\cos(\theta)^2+r^2} \\ 0 & \frac{a^2\cos(\theta)^2+r^2}{r^2-2mr+e^2+a^2} & 0 & 0 \\ 0 & 0 & a^2\cos(\theta)^2+r^2 & 0 \\ \frac{a(e^2-2mr)\sin(\theta)^2}{a^2\cos(\theta)^2+r^2} & 0 & 0 & \frac{\sin(\theta)^2\left((r^2+a^2)^2-a^2(r^2-2mr+e^2+a^2)\sin(\theta)^2\right)}{a^2\cos(\theta)^2+r^2} \end{bmatrix}, 22, \textit{kerr\_newman}\right]$$

```
(%o10) done
```

```
(%i11) lcs_matrix():=block([h,i,j,nn,nuova],
         nn:length(ct_coords),
         lcs_m:makelist(0,h,1,nn),
         for h:1 thru nn do(
         nuova:trigsimp(genmatrix(lambda([i,j],lcs[i,j,h]),nn,nn,1,1)),
         lcs_m[h]:ratsimp(nuova)
         ),
         "Ha aggiornato lcs_m")$
```

```
(%i12) mcs_matrix():=block([h,i,j,nn,nuova],
         nn:length(ct_coords),
         mcs_m:makelist(0,h,1,nn),
         for h:1 thru nn do(
         nuova:trigsimp(genmatrix(lambda([i,j],mcs[i,j,h]),nn,nn,1,1)),
         mcs_m[h]:ratsimp(nuova)
         ),
         "Ha aggiornato mcs_m")$
```

```
(%i13) prima_specie(oa,ob,printo):=block([j,nct,h,a,b],
         for j:oa thru min(ob,length(opzioni)) do (
            print(["Simboli di Christoffel prima specie",opzioni[j]]),
            ct_coordsys(opzioni[j]),
            cmetric(),
            if  printo[1] then
                print([lg,"= Tensore metrico g22"]),
            nct:length(ct_coords),
            christof(false),
            lcs_matrix(),
            if printo[1] then (
                print([lcs_m[1],"= H20_22:",ct_coords[1]]),
                print([lcs_m[2],"= H21_22:",ct_coords[2]]),
                if nct>2 then
                print([lcs_m[3],"= H22_22:",ct_coords[3]]),
                if nct>3 then
                print([lcs_m[4],"= H23_22:",ct_coords[4]])
                )
            else (
               print("Componenti del tensore metrico"),
               for a:1 thru nct do
                     for b:a thru nct do if lg[a,b]#0 then
                     if printo[length(printo)] then print( [lg[a,b],"=",
                     ct_coords[a],ct_coords[b]] ),
               print("Simboli di Christoffel di prima specie"),
               for h:1 thru nct do if printo[h+1] then (
                  print(["Per la componente ",ct_coords[h]]),
                  for a:1 thru nct do
                     for b:a thru nct do if lcs_m[h][a,b]#0 then
                     print( [lcs_m[h][a,b],"=",
                     ct_coords[h],ct_coords[a],ct_coords[b]] )
                  )
               )
            )
         )$
```

```
(%i14) seconda_specie(oa,ob,printo):=block([j,nct,h,a,b],
         for j:oa thru min(ob,length(opzioni)) do (
            print(["Simboli di Christoffel seconda specie",opzioni[j]]),
            ct_coordsys(opzioni[j]),
            cmetric(),
            if  printo[1] then
                print([lg,"= Tensore metrico g22"]),
            nct:length(ct_coords),
            christof(false),
            mcs_matrix(),
            if printo[1] then (
                print([mcs_m[1],"= H10_22:",ct_coords[1]]),
                print([mcs_m[2],"= H11_22:",ct_coords[2]]),
                if nct>2 then
                print([mcs_m[3],"= H12_22:",ct_coords[3]]),
                if nct>3 then
                print([mcs_m[4],"= H13_22:",ct_coords[4]])
                )
            else (
               print("Componenti del tensore metrico"),
               for a:1 thru nct do
                     for b:a thru nct do if lg[a,b]#0 then
                     if printo[length(printo)] then print( [lg[a,b],"=",
                     ct_coords[a],ct_coords[b]] ),
               print("Simboli di Christoffel di seconda specie"),
               for h:1 thru nct do if printo[h+1] then (
                  print(["Per la componente ",ct_coords[h]]),
                  for a:1 thru nct do
                     for b:a thru nct do if mcs_m[h][a,b]#0 then
                     print( [mcs_m[h][a,b],"=",
                     ct_coords[h],ct_coords[a],ct_coords[b]] )
                  )
                )
            )
         )$

(%i15) si:[true];
(%o15) [true]

(%i16) no:[false,true,true,true,true,true];
(%o16) [false,true,true,true,true,true]

(%i17) no1:[false,true,false,false,false,true];
(%o17) [false,true,false,false,false,true]

(%i18) no2:[false,false,true,false,false,false];
(%o18) [false,false,true,false,false,false]

(%i19) no3:[false,false,false,true,false,false];
(%o19) [false,false,false,true,false,false]
```

```
(%i20) no4:[false,false,false,false,true,false];
(%o20) [false,false,false,false,true,false]
```

Elenco dei simboli di Christoffel di prima specie.

```
(%i21) prima_specie(1,1,no);
[Simboli di Christoffel prima specie,cartesian2d]
Componenti del tensore metrico
[1,=,x,x]
[1,=,y,y]
Simboli di Christoffel di prima specie
[Per la componente ,x]
[Per la componente ,y]
(%o21) done
```

```
(%i22) prima_specie(2,2,no);
[Simboli di Christoffel prima specie,polar]
Componenti del tensore metrico
[1,=,r,r]
[r^2,=,ϕ,ϕ]
Simboli di Christoffel di prima specie
[Per la componente ,r]
[-r,=,r,ϕ,ϕ]
[Per la componente ,ϕ]
[r,=,ϕ,r,ϕ]
(%o22) done
```

```
(%i23) prima_specie(3,3,s1);
[Simboli di Christoffel prima specie,elliptic]
(%o23) done
```

```
(%i24) prima_specie(4,4,s1);
[Simboli di Christoffel prima specie,confocalelliptic]
(%o24) done
```

```
(%i25) prima_specie(5,5,s1);
[Simboli di Christoffel prima specie,bipolar]
(%o25) done
```

```
(%i26) prima_specie(6,6,si);
[Simboli di Christoffel prima specie,parabolic]
[matrix([v^2+u^2,0],[0,v^2+u^2]),= Tensore metrico g22]
[matrix([u,v],[v,-u]),= H20_22:,u]
[matrix([-v,u],[u,v]),= H21_22:,v]
(%o26) done
```

```
(%i27) prima_specie(7,7,si);
[Simboli di Christoffel prima specie,cartesian3d]
[matrix([1,0,0],[0,1,0],[0,0,1]),= Tensore metrico g22]
[matrix([0,0,0],[0,0,0],[0,0,0]),= H20_22:,x]
[matrix([0,0,0],[0,0,0],[0,0,0]),= H21_22:,y]
[matrix([0,0,0],[0,0,0],[0,0,0]),= H22_22:,z]
(%o27) done
```

(%i28) prima_specie(8,8,si);

[Simboli di Christoffel prima specie,polarcylindrical]

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, = \text{Tensore metrico g22}\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & -r & 0 \\ 0 & 0 & 0 \end{pmatrix}, = \text{H20\_22:,r}\right]$$

$$\left[\begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, = \text{H21\_22:,}\theta\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, = \text{H22\_22:,z}\right]$$

(%o28) done

(%i29) prima_specie(9,9,si);

[Simboli di Christoffel prima specie,paraboloidal]

$$\left[\begin{pmatrix} u^2 v^2 & 0 & 0 \\ 0 & v^2+u^2 & 0 \\ 0 & 0 & v^2+u^2 \end{pmatrix}, = \text{Tensore metrico g22}\right]$$

$$\left[\begin{pmatrix} u v^2 & u^2 v & 0 \\ u^2 v & -u & 0 \\ 0 & 0 & -u \end{pmatrix}, = \text{H20\_22:,u}\right]$$

$$\left[\begin{pmatrix} -u^2 v & u & 0 \\ u & v & 0 \\ 0 & 0 & -v \end{pmatrix}, = \text{H21\_22:,v}\right]$$

$$\left[\begin{pmatrix} 0 & 0 & u \\ 0 & 0 & v \\ u & v & 0 \end{pmatrix}, = \text{H22\_22:,}\phi\right]$$

(%o29) done

```
(%i30) prima_specie(10,10,no);
```

[Simboli di Christoffel prima specie,conical]

Componenti del tensore metrico

$$\left[\frac{(v-u)(v+u)w^2}{(u-e)(u+e)(u-f)(u+f)},=,u,u\right]$$

$$\left[\frac{(u-v)(v+u)w^2}{(v-e)(v+e)(v-f)(v+f)},=,v,v\right]$$

$$[1,=,w,w]$$

Simboli di Christoffel di prima specie

[Per la componente ,u]

$$\left[-\frac{((2u^3+(-f^2-e^2)u)v^2-u^5+e^2f^2u)w^2}{u^8+(-2f^2-2e^2)u^6+(f^4+4e^2f^2+e^4)u^4+(-2e^2f^4-2e^4f^2)u^2+e^4f^4},=,u,u,u\right]$$

$$\left[\frac{vw^2}{u^4+(-f^2-e^2)u^2+e^2f^2},=,u,u,v\right]$$

$$\left[\frac{(v^2-u^2)w}{u^4+(-f^2-e^2)u^2+e^2f^2},=,u,u,w\right]$$

$$\left[-\frac{uw^2}{v^4+(-f^2-e^2)v^2+e^2f^2},=,u,v,v\right]$$

[Per la componente ,v]

$$\left[-\frac{vw^2}{u^4+(-f^2-e^2)u^2+e^2f^2},=,v,u,u\right]$$

$$\left[\frac{uw^2}{v^4+(-f^2-e^2)v^2+e^2f^2},=,v,u,v\right]$$

$$\left[\frac{(v^5-2u^2v^3+((f^2+e^2)u^2-e^2f^2)v)w^2}{v^8+(-2f^2-2e^2)v^6+(f^4+4e^2f^2+e^4)v^4+(-2e^2f^4-2e^4f^2)v^2+e^4f^4},=,v,v,v\right]$$

$$\left[-\frac{(v^2-u^2)w}{v^4+(-f^2-e^2)v^2+e^2f^2},=,v,v,w\right]$$

[Per la componente ,w]

$$\left[-\frac{(v^2-u^2)w}{u^4+(-f^2-e^2)u^2+e^2f^2},=,w,u,u\right]$$

$$\left[\frac{(v^2-u^2)w}{v^4+(-f^2-e^2)v^2+e^2f^2},=,w,v,v\right]$$

```
(%o30) done
```

```
(%i31) prima_specie(11,11,no1);
```

[Simboli di Christoffel prima specie,toroidal]

Componenti del tensore metrico

$$\left[\frac{e^2\sinh(v)^2}{(\cosh(v)-\cos(u))^2},=,u,u\right]$$

$$\left[\frac{e^2}{(\cosh(v)-\cos(u))^2},=,v,v\right]$$

$$\left[\frac{e^2}{(\cosh(v)-\cos(u))^2},=,\phi,\phi\right]$$

Simboli di Christoffel di prima specie

[Per la componente ,u]

$$\left[-\frac{e^2\sin(u)\cosh(v)^2-e^2\sin(u)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,u,u,u\right]$$

$$\left[-\frac{(e^2\cos(u)\cosh(v)-e^2)\sinh(v)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,u,u,v\right]$$

$$\left[\frac{e^2\sin(u)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,u,v,v\right]$$

$$\left[\frac{e^2\sin(u)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,u,\phi,\phi\right]$$

(%o31) *done*

```
(%i32) prima_specie(11,11,no2);
```

[Simboli di Christoffel prima specie,toroidal]

Componenti del tensore metrico

Simboli di Christoffel di prima specie

[Per la componente ,v]

$$\left[\frac{(e^2\cos(u)\cosh(v)-e^2)\sinh(v)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,v,u,u\right]$$

$$\left[-\frac{e^2\sin(u)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,v,u,v\right]$$

$$\left[-\frac{e^2\sinh(v)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,v,v,v\right]$$

$$\left[\frac{e^2\sinh(v)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,v,\phi,\phi\right]$$

(%o32) *done*

```
(%i33) prima_specie(11,11,no3);
[Simboli di Christoffel prima specie,toroidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
[Per la componente ,ϕ]
```

$$\left[-\frac{e^2\sin(u)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,\phi,u,\phi\right]$$

$$\left[-\frac{e^2\sinh(v)}{\cosh(v)^3-3\cos(u)\cosh(v)^2+3\cos(u)^2\cosh(v)-\cos(u)^3},=,\phi,v,\phi\right]$$

```
(%o33) done

(%i34) prima_specie(11,11,no4);
[Simboli di Christoffel prima specie,toroidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
(%o34) done

(%i35) prima_specie(12,12,si);
[Simboli di Christoffel prima specie,spherical]
```

$$\left[\begin{bmatrix}1&0&0\\0&r^2&0\\0&0&r^2\sin(\theta)^2\end{bmatrix},=\text{ Tensore metrico g22}\right]$$

$$\left[\begin{bmatrix}0&0&0\\0&-r&0\\0&0&-r\sin(\theta)^2\end{bmatrix},=\text{ H20\_22:},r\right]$$

$$\left[\begin{bmatrix}0&r&0\\r&0&0\\0&0&-r^2\cos(\theta)\sin(\theta)\end{bmatrix},=\text{ H21\_22:},\theta\right]$$

$$\left[\begin{bmatrix}0&0&r\sin(\theta)^2\\0&0&r^2\cos(\theta)\sin(\theta)\\r\sin(\theta)^2&r^2\cos(\theta)\sin(\theta)&0\end{bmatrix},=\text{ H22\_22:},\phi\right]$$

```
(%o35) done
```

```
(%i36) prima_specie(13,13,no1);
```
[Simboli di Christoffel prima specie,oblatespheroidal]
Componenti del tensore metrico
$$[e^2(\sin(v)^2+\sinh(u)^2),=,u,u]$$
$$[e^2(\sin(v)^2+\sinh(u)^2),=,v,v]$$
$$[e^2\cosh(u)^2\cos(v)^2,=,\phi,\phi]$$
Simboli di Christoffel di prima specie
[Per la componente ,u]
$$[e^2\cosh(u)\sinh(u),=,u,u,u]$$
$$[e^2\cos(v)\sin(v),=,u,u,v]$$
$$[-e^2\cosh(u)\sinh(u),=,u,v,v]$$
$$[-e^2\cosh(u)\sinh(u)\cos(v)^2,=,u,\phi,\phi]$$
(%o36) done

```
(%i37) prima_specie(13,13,no2);
```
[Simboli di Christoffel prima specie,oblatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
[Per la componente ,v]
$$[-e^2\cos(v)\sin(v),=,v,u,u]$$
$$[e^2\cosh(u)\sinh(u),=,v,u,v]$$
$$[e^2\cos(v)\sin(v),=,v,v,v]$$
$$[e^2\cosh(u)^2\cos(v)\sin(v),=,v,\phi,\phi]$$
(%o37) done

```
(%i38) prima_specie(13,13,no3);
```
[Simboli di Christoffel prima specie,oblatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
[Per la componente ,ϕ]
$$[e^2\cosh(u)\sinh(u)\cos(v)^2,=,\phi,u,\phi]$$
$$[-e^2\cosh(u)^2\cos(v)\sin(v),=,\phi,v,\phi]$$
(%o38) done

```
(%i39) prima_specie(13,13,no4);
```
[Simboli di Christoffel prima specie,oblatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
(%o39) done

(%i40) prima_specie(14,14,si);

[Simboli di Christoffel prima specie,oblatespheroidalsqrt]

$$\left[\begin{matrix}\frac{e^2\,(u^2-v^2)}{u^2-1} & 0 & 0\\ 0 & \frac{e^2\,(u^2-v^2)}{u^2-1} & 0\\ 0 & 0 & e^2\,u^2\,v^2\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}\frac{e^2\,u\,v^2-e^2\,u}{u^4-2\,u^2+1} & -\frac{e^2\,v}{u^2-1} & 0\\ -\frac{e^2\,v}{u^2-1} & -\frac{e^2\,u\,v^2-e^2\,u}{u^4-2\,u^2+1} & 0\\ 0 & 0 & -e^2\,u\,v^2\end{matrix}\right], = H20\_22:,u]$$

$$\left[\begin{matrix}\frac{e^2\,v}{u^2-1} & \frac{e^2\,u\,v^2-e^2\,u}{u^4-2\,u^2+1} & 0\\ \frac{e^2\,u\,v^2-e^2\,u}{u^4-2\,u^2+1} & -\frac{e^2\,v}{u^2-1} & 0\\ 0 & 0 & -e^2\,u^2\,v\end{matrix}\right], = H21\_22:,v]$$

$$\left[\begin{matrix}0 & 0 & e^2\,u\,v^2\\ 0 & 0 & e^2\,u^2\,v\\ e^2\,u\,v^2 & e^2\,u^2\,v & 0\end{matrix}\right], = H22\_22:,\phi]$$

(%o40) done

```
(%i41) prima_specie(15,15,si);
```

[Simboli di Christoffel prima specie,prolatespheroidal]

$$\left[\begin{matrix} e^2(\sin(v)^2+\sinh(u)^2) & 0 & 0 \\ 0 & e^2(\sin(v)^2+\sinh(u)^2) & 0 \\ 0 & 0 & e^2\sinh(u)^2\sin(v)^2 \end{matrix}\right],$$

= Tensore metrico g22]

$$\left[\begin{matrix} e^2\cosh(u)\sinh(u) & e^2\cos(v)\sin(v) & 0 \\ e^2\cos(v)\sin(v) & -e^2\cosh(u)\sinh(u) & 0 \\ 0 & 0 & -e^2\cosh(u)\sinh(u)\sin(v)^2 \end{matrix}\right], = H20\_22:, u]$$

$$\left[\begin{matrix} -e^2\cos(v)\sin(v) & e^2\cosh(u)\sinh(u) & 0 \\ e^2\cosh(u)\sinh(u) & e^2\cos(v)\sin(v) & 0 \\ 0 & 0 & -e^2\sinh(u)^2\cos(v)\sin(v) \end{matrix}\right], = H21\_22:, v]$$

$$\left[\begin{matrix} 0 & 0 & e^2\cosh(u)\sinh(u)\sin(v)^2 \\ 0 & 0 & e^2\sinh(u)^2\cos(v)\sin(v) \\ e^2\cosh(u)\sinh(u)\sin(v)^2 & e^2\sinh(u)^2\cos(v)\sin(v) & 0 \end{matrix}\right],$$

= H22_22:, ϕ]

(%o41) done

```
(%i42) prima_specie(16,16,si);
```

[Simboli di Christoffel prima specie,prolatespheroidalsqrt]

$$\left[\begin{matrix}\frac{e^2\,(v^2-u^2)}{1-u^2} & 0 & 0\\ 0 & \frac{e^2\,(v^2-u^2)}{v^2-1} & 0\\ 0 & 0 & e^2\,(1-u^2)\,(v^2-1)\end{matrix}\right], = \textit{Tensore metrico g22}]$$

$$\left[\begin{matrix}\frac{e^2\,u\,v^2-e^2\,u}{u^4-2\,u^2+1} & -\frac{e^2\,v}{u^2-1} & 0\\ -\frac{e^2\,v}{u^2-1} & \frac{e^2\,u}{v^2-1} & 0\\ 0 & 0 & e^2\,u\,v^2-e^2\,u\end{matrix}\right], = H20\_22:,u]$$

$$\left[\begin{matrix}\frac{e^2\,v}{u^2-1} & -\frac{e^2\,u}{v^2-1} & 0\\ -\frac{e^2\,u}{v^2-1} & \frac{(e^2\,u^2-e^2)\,v}{v^4-2\,v^2+1} & 0\\ 0 & 0 & (e^2\,u^2-e^2)\,v\end{matrix}\right], = H21\_22:,v]$$

$$\left[\begin{matrix}0 & 0 & e^2\,u-e^2\,u\,v^2\\ 0 & 0 & (e^2-e^2\,u^2)\,v\\ e^2\,u-e^2\,u\,v^2 & (e^2-e^2\,u^2)\,v & 0\end{matrix}\right], = H22\_22:,\phi]$$

```
(%o42) done
```

```
(%i43) prima_specie(17,17,no1);
```

[Simboli di Christoffel prima specie,ellipsoidal]

Componenti del tensore metrico

$$[(b^2\sin(\phi)^2+a^2\cos(\phi)^2)\sin(\theta)^2+c^2\cos(\theta)^2,=,r,r]$$

$$[(b^2\sin(\phi)^2+a^2\cos(\phi)^2-c^2)r\cos(\theta)\sin(\theta),=,r,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)r\sin(\theta)^2,=,r,\phi]$$

$$[r^2\left(c^2\sin(\theta)^2+(b^2\sin(\phi)^2+a^2\cos(\phi)^2)\cos(\theta)^2\right),=,\theta,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)r^2\cos(\theta)\sin(\theta),=,\theta,\phi]$$

$$[(a^2\sin(\phi)^2+b^2\cos(\phi)^2)r^2\sin(\theta)^2,=,\phi,\phi]$$

Simboli di Christoffel di prima specie

[Per la componente ,r]

$$[(b^2\sin(\phi)^2+a^2\cos(\phi)^2-c^2)\cos(\theta)\sin(\theta),=,r,r,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)\sin(\theta)^2,=,r,r,\phi]$$

$$[(-b^2\sin(\phi)^2-a^2\cos(\phi)^2+c^2)r\sin(\theta)^2-c^2\,r,=,r,\theta,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)r\cos(\theta)\sin(\theta),=,r,\theta,\phi]$$

$$[(-b^2\sin(\phi)^2-a^2\cos(\phi)^2)r\sin(\theta)^2,=,r,\phi,\phi]$$

(%o43) *done*

```
(%i44) prima_specie(17,17,no2);
```

[Simboli di Christoffel prima specie,ellipsoidal]

Componenti del tensore metrico

Simboli di Christoffel di prima specie

[Per la componente ,θ]

$$[c^2\,r\sin(\theta)^2+(b^2\sin(\phi)^2+a^2\cos(\phi)^2)r\cos(\theta)^2,=,\theta,r,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)r\cos(\theta)\sin(\theta),=,\theta,r,\phi]$$

$$[(-b^2\sin(\phi)^2-a^2\cos(\phi)^2+c^2)r^2\cos(\theta)\sin(\theta),=,\theta,\theta,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)r^2\cos(\theta)^2,=,\theta,\theta,\phi]$$

$$[(-b^2\sin(\phi)^2-a^2\cos(\phi)^2)r^2\cos(\theta)\sin(\theta),=,\theta,\phi,\phi]$$

(%o44) *done*

```
(%i45) prima_specie(17,17,no3);
```

[Simboli di Christoffel prima specie,ellipsoidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
[Per la componente ,ϕ]

$$[(b^2-a^2)\cos(\phi)\sin(\phi)\,r\cos(\theta)\sin(\theta),=,\phi,r,\theta]$$

$$[(a^2\sin(\phi)^2+b^2\cos(\phi)^2)\,r\sin(\theta)^2,=,\phi,r,\phi]$$

$$[(a^2-b^2)\cos(\phi)\sin(\phi)\,r^2\sin(\theta)^2,=,\phi,\theta,\theta]$$

$$[(a^2\sin(\phi)^2+b^2\cos(\phi)^2)\,r^2\cos(\theta)\sin(\theta),=,\phi,\theta,\phi]$$

$$[(a^2-b^2)\cos(\phi)\sin(\phi)\,r^2\sin(\theta)^2,=,\phi,\phi,\phi]$$

(%o45) done

```
(%i46) prima_specie(17,17,no4);
```

[Simboli di Christoffel prima specie,ellipsoidal]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
(%o46) done

```
(%i47) prima_specie(18,18,si);
[Simboli di Christoffel prima specie,cartesian4d]
                [1  0  0  0]
                [           ]
                [0  1  0  0]
[                           ,= Tensore metrico g22]
                [           ]
                [0  0  1  0]
                [           ]
                [0  0  0  1]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
[                           ,= H20_22:,x]
                [           ]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
[                           ,= H21_22:,y]
                [           ]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
[                           ,= H22_22:,z]
                [           ]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
[                           ,= H23_22:,t]
                [           ]
                [0  0  0  0]
                [           ]
                [0  0  0  0]
(%o47) done
```

```
(%i48) prima_specie(19,19,si);
[Simboli di Christoffel prima specie,spherical4d]
```

$$\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 & 0 \\ 0 & 0 & 0 & \sin(\eta)^2 r^2 \sin(\theta)^2 \end{pmatrix}, = \text{Tensore metrico g22}\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & -r\sin(\theta)^2 & 0 \\ 0 & 0 & 0 & -\sin(\eta)^2 r \sin(\theta)^2 \end{pmatrix}, = H20\_22:, r\right]$$

$$\left[\begin{pmatrix} 0 & r & 0 & 0 \\ r & 0 & 0 & 0 \\ 0 & 0 & -r^2\cos(\theta)\sin(\theta) & 0 \\ 0 & 0 & 0 & -\sin(\eta)^2 r^2 \cos(\theta)\sin(\theta) \end{pmatrix}, = H21\_22:, \theta\right]$$

$$\left[\begin{pmatrix} 0 & 0 & r\sin(\theta)^2 & 0 \\ 0 & 0 & r^2\cos(\theta)\sin(\theta) & 0 \\ r\sin(\theta)^2 & r^2\cos(\theta)\sin(\theta) & 0 & 0 \\ 0 & 0 & 0 & -\cos(\eta)\sin(\eta) r^2 \sin(\theta)^2 \end{pmatrix}, = H22\_22:, \eta\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & \sin(\eta)^2 r \sin(\theta)^2 \\ 0 & 0 & 0 & \sin(\eta)^2 r^2 \cos(\theta)\sin(\theta) \\ 0 & 0 & 0 & \cos(\eta)\sin(\eta) r^2 \sin(\theta)^2 \\ \sin(\eta)^2 r \sin(\theta)^2 & \sin(\eta)^2 r^2 \cos(\theta)\sin(\theta) & \cos(\eta)\sin(\eta) r^2 \sin(\theta)^2 & 0 \end{pmatrix}, = H23\_22:, \phi\right]$$

```
(%o48) done
```

```
(%i49) prima_specie(20,20,si);
```

[Simboli di Christoffel prima specie,exteriorschwarzschild]

$$\left[\begin{pmatrix} \frac{2\,m-r}{r} & 0 & 0 & 0 \\ 0 & \frac{r}{r-2\,m} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)^2 \end{pmatrix}, = \textit{Tensore metrico g22}\right]$$

$$\left[\begin{pmatrix} 0 & -\frac{m}{r^2} & 0 & 0 \\ -\frac{m}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, = \textit{H20\_22:,t}\right]$$

$$\left[\begin{pmatrix} \frac{m}{r^2} & 0 & 0 & 0 \\ 0 & -\frac{m}{r^2-4\,m\,r+4\,m^2} & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & 0 & 0 & -r\sin(\theta)^2 \end{pmatrix}, = \textit{H21\_22:,r}\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & 0 & -r^2\cos(\theta)\sin(\theta) \end{pmatrix}, = \textit{H22\_22:,}\theta\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r\sin(\theta)^2 \\ 0 & 0 & 0 & r^2\cos(\theta)\sin(\theta) \\ 0 & r\sin(\theta)^2 & r^2\cos(\theta)\sin(\theta) & 0 \end{pmatrix}, = \textit{H23\_22:,}\phi\right]$$

```
(%o49) done
```

```
(%i50) prima_specie(21,21,si);
```

[Simboli di Christoffel prima specie,interiorschwarzschild]

$$\left[\begin{pmatrix} -\frac{t}{2\,m-t} & 0 & 0 & 0 \\ 0 & \frac{2\,m-t}{t} & 0 & 0 \\ 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & t^2\sin(u)^2 \end{pmatrix}, = \textit{Tensore metrico g22}\right]$$

$$\left[\begin{pmatrix} -\frac{m}{t^2-4\,m\,t+4\,m^2} & 0 & 0 & 0 \\ 0 & \frac{m}{t^2} & 0 & 0 \\ 0 & 0 & -t & 0 \\ 0 & 0 & 0 & -t\sin(u)^2 \end{pmatrix}, = \textit{H20\_22:,t}\right]$$

$$\left[\begin{pmatrix} 0 & -\frac{m}{t^2} & 0 & 0 \\ -\frac{m}{t^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, = \textit{H21\_22:,z}\right]$$

$$\left[\begin{pmatrix} 0 & 0 & t & 0 \\ 0 & 0 & 0 & 0 \\ t & 0 & 0 & 0 \\ 0 & 0 & 0 & -t^2\cos(u)\sin(u) \end{pmatrix}, = \textit{H22\_22:,u}\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & t\sin(u)^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t^2\cos(u)\sin(u) \\ t\sin(u)^2 & 0 & t^2\cos(u)\sin(u) & 0 \end{pmatrix}, = \textit{H23\_22:,v}\right]$$

(%o50) *done*

```
(%i51) prima_specie(22,22,no1);
```

[Simboli di Christoffel prima specie,kerr_newman]

Componenti del tensore metrico

$$\left[\frac{a^2\sin(\theta)^2-r^2+2\,m\,r-e^2-a^2}{a^2\cos(\theta)^2+r^2},=,ct,ct\right]$$

$$\left[\frac{a\,(e^2-2\,m\,r)\sin(\theta)^2}{a^2\cos(\theta)^2+r^2},=,ct,\phi\right]$$

$$\left[\frac{a^2\cos(\theta)^2+r^2}{r^2-2\,m\,r+e^2+a^2},=,r,r\right]$$

$$\left[a^2\cos(\theta)^2+r^2,=,\theta,\theta\right]$$

$$\left[\frac{\sin(\theta)^2\,((r^2+a^2)^2-a^2\,(r^2-2\,m\,r+e^2+a^2)\sin(\theta)^2)}{a^2\cos(\theta)^2+r^2},=,\phi,\phi\right]$$

Simboli di Christoffel di prima specie

[Per la componente ,ct]

$$\left[\frac{a^2\,m\cos(\theta)^2-m\,r^2+e^2\,r}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,ct,ct,r\right]$$

$$\left[\frac{(2\,a^2\,m\,r-a^2\,e^2)\cos(\theta)\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,ct,ct,\theta\right]$$

$$\left[\frac{a^3\,m\cos(\theta)^4+(-a\,m\,r^2+a\,e^2\,r-a^3\,m)\cos(\theta)^2+a\,m\,r^2-a\,e^2\,r}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,ct,r,\phi\right]$$

$$\left[-\frac{(2\,a\,m\,r^3-a\,e^2\,r^2+2\,a^3\,m\,r-a^3\,e^2)\cos(\theta)\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,ct,\theta,\phi\right]$$

(%o51) *done*

```
(%i52) prima_specie(22,22,no2);
[Simboli di Christoffel prima specie,kerr_newman]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
[Per la componente ,r]
```

$$\left[\frac{a^2\,r\sin(\theta)^2+(a^2\,r-a^2\,m)\cos(\theta)^2+m\,r^2+(-e^2-a^2)\,r}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,r,ct,ct\right]$$

$$\left[\frac{(a^3\,m\cos(\theta)^2-a\,m\,r^2+a\,e^2\,r)\sin(\theta)^2}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,r,ct,\phi\right]$$

$$\left[-\frac{(a^2\,r-a^2\,m)\cos(\theta)^2+m\,r^2+(-e^2-a^2)\,r}{r^4-4\,m\,r^3+(4\,m^2+2\,e^2+2\,a^2)\,r^2+(-4\,e^2-4\,a^2)\,m\,r+e^4+2\,a^2\,e^2+a^4},=,r,r,r\right]$$

$$\left[-\frac{a^2\cos(\theta)\sin(\theta)}{r^2-2\,m\,r+e^2+a^2},=,r,r,\theta\right]$$

$$[-r,=,r,\theta,\theta]$$

$$[(((a^4\,r-a^4\,m)\cos(\theta)^2+a^2\,m\,r^2+(-a^2\,e^2-a^4)\,r)\sin(\theta)^4+((-2\,a^2\,r^3-2\,a^4\,r)\cos(\theta)^2-r^5+a^4\,r)\sin(\theta)^2)/(a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4),=,r,\phi,\phi]$$

```
(%o52) done

(%i53) prima_specie(22,22,no3);
[Simboli di Christoffel prima specie,kerr_newman]
Componenti del tensore metrico
Simboli di Christoffel di prima specie
[Per la componente ,θ]
```

$$\left[-\frac{(2\,a^2\,m\,r-a^2\,e^2)\cos(\theta)\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,\theta,ct,ct\right]$$

$$\left[\frac{(2\,a\,m\,r^3-a\,e^2\,r^2+2\,a^3\,m\,r-a^3\,e^2)\cos(\theta)\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,\theta,ct,\phi\right]$$

$$\left[\frac{a^2\cos(\theta)\sin(\theta)}{r^2-2\,m\,r+e^2+a^2},=,\theta,r,r\right]$$

$$[r,=,\theta,r,\theta]$$

$$[-a^2\cos(\theta)\sin(\theta),=,\theta,\theta,\theta]$$

$$[-((a^4\,r^2-2\,a^4\,m\,r+a^4\,e^2+a^6)\cos(\theta)^5+(2\,a^2\,r^4-4\,a^2\,m\,r^3+(2\,a^2\,e^2+2\,a^4)\,r^2)\cos(\theta)^3+(r^6+a^2\,r^4+4\,a^2\,m\,r^3-2\,a^2\,e^2\,r^2+2\,a^4\,m\,r-a^4\,e^2)\cos(\theta))\sin(\theta))/(a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4),=,\theta,\phi,\phi]$$

```
(%o53) done
```

```
(%i54) prima_specie(22,22,no4);
```

[Simboli di Christoffel prima specie,kerr_newman]

Componenti del tensore metrico

Simboli di Christoffel di prima specie

[Per la componente ,ϕ]

$$\left[\frac{a^3\,m\cos(\theta)^4+(-a\,m\,r^2+a\,e^2\,r-a^3\,m)\cos(\theta)^2+a\,m\,r^2-a\,e^2\,r}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,\phi,ct,r\right]$$

$$\left[-\frac{(2\,a\,m\,r^3-a\,e^2\,r^2+2\,a^3\,m\,r-a^3\,e^2)\cos(\theta)\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,\phi,ct,\theta\right]$$

$$[-((a^4\,r-a^4\,m)\cos(\theta)^6+(2\,a^2\,r^3+a^2\,m\,r^2+(-a^2\,e^2-a^4)\,r+2\,a^4\,m)\cos(\theta)^4+(r^5-2\,a^2\,r^3-2\,a^2\,m\,r^2+2\,a^2\,e^2\,r-a^4\,m)\cos(\theta)^2-r^5+a^2\,m\,r^2-a^2\,e^2\,r)/(a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4),=,\phi,r,\phi]$$

$$[(((a^4\,r^2-2\,a^4\,m\,r+a^4\,e^2+a^6)\cos(\theta)^5+(2\,a^2\,r^4-4\,a^2\,m\,r^3+(2\,a^2\,e^2+2\,a^4)\,r^2)\cos(\theta)^3+(r^6+a^2\,r^4+4\,a^2\,m\,r^3-2\,a^2\,e^2\,r^2+2\,a^4\,m\,r-a^4\,e^2)\cos(\theta))\sin(\theta))/(a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4),=,\phi,\theta,\phi]$$

(%o54) done

Elenco dei simboli di Christoffel di seconda specie...

```
(%i55) seconda_specie(1,1,si);
```

[Simboli di Christoffel seconda specie,cartesian2d]

$$\left[\begin{bmatrix}1&0\\0&1\end{bmatrix},=\text{Tensore metrico g22}\right]$$

$$\left[\begin{bmatrix}0&0\\0&0\end{bmatrix},=H10\_22:,x\right]$$

$$\left[\begin{bmatrix}0&0\\0&0\end{bmatrix},=H11\_22:,y\right]$$

(%o55) done

(%i56) seconda_specie(2,2,si);

[Simboli di Christoffel seconda specie,polar]

$$\left[\begin{matrix}1 & 0\\ 0 & r^2\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}0 & 0\\ 0 & -r\end{matrix}\right], = \text{H10\_22:,r}]$$

$$\left[\begin{matrix}0 & \frac{1}{r}\\ \frac{1}{r} & 0\end{matrix}\right], = \text{H11\_22:,}\phi]$$

(%o56) done

(%i57) seconda_specie(3,3,si);

[Simboli di Christoffel seconda specie,elliptic]

$$\left[\begin{matrix}e^2\left(\cosh(u)^2-\cos(v)^2\right) & 0\\ 0 & e^2\left(\cosh(u)^2-\cos(v)^2\right)\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2} & \frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2}\\ \frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2} & -\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2}\end{matrix}\right], = \text{H10\_22:,u}]$$

$$\left[\begin{matrix}-\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2} & \frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2}\\ \frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2} & \frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2}\end{matrix}\right], = \text{H11\_22:,v}]$$

(%o57) done

(%i58) seconda_specie(4,4,si);

[Simboli di Christoffel seconda specie,confocalelliptic]

$$\left[\begin{matrix}\frac{e^2\,(u^2-v^2)}{u^2-1} & 0\\ 0 & \frac{e^2\,(v^2-u^2)}{v^2-1}\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}-\frac{u\,v^2-u}{(u^2-1)\,v^2-u^4+u^2} & \frac{v}{v^2-u^2}\\ \frac{v}{v^2-u^2} & -\frac{u^3-u}{v^4+(-u^2-1)\,v^2+u^2}\end{matrix}\right], = H10\_22:,u]$$

$$\left[\begin{matrix}\frac{v^3-v}{(u^2-1)\,v^2-u^4+u^2} & -\frac{u}{v^2-u^2}\\ -\frac{u}{v^2-u^2} & \frac{(u^2-1)\,v}{v^4+(-u^2-1)\,v^2+u^2}\end{matrix}\right], = H11\_22:,v]$$

(%o58) done

(%i59) seconda_specie(5,5,si);

[Simboli di Christoffel seconda specie,bipolar]

$$\left[\begin{matrix}\frac{e^2}{(\cosh(v)-\cos(u))^2} & 0\\ 0 & \frac{e^2}{(\cosh(v)-\cos(u))^2}\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}-\frac{\sin(u)}{\cosh(v)-\cos(u)} & -\frac{\sinh(v)}{\cosh(v)-\cos(u)}\\ -\frac{\sinh(v)}{\cosh(v)-\cos(u)} & \frac{\sin(u)}{\cosh(v)-\cos(u)}\end{matrix}\right], = H10\_22:,u]$$

$$\left[\begin{matrix}\frac{\sinh(v)}{\cosh(v)-\cos(u)} & -\frac{\sin(u)}{\cosh(v)-\cos(u)}\\ -\frac{\sin(u)}{\cosh(v)-\cos(u)} & -\frac{\sinh(v)}{\cosh(v)-\cos(u)}\end{matrix}\right], = H11\_22:,v]$$

(%o59) done

```
(%i60) seconda_specie(6,6,si);
```

[Simboli di Christoffel seconda specie,parabolic]

$$\left[\begin{matrix} v^2+u^2 & 0 \\ 0 & v^2+u^2 \end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix} \frac{u}{v^2+u^2} & \frac{v}{v^2+u^2} \\ \frac{v}{v^2+u^2} & -\frac{u}{v^2+u^2} \end{matrix}\right], = \text{H10\_22:,u}]$$

$$\left[\begin{matrix} -\frac{v}{v^2+u^2} & \frac{u}{v^2+u^2} \\ \frac{u}{v^2+u^2} & \frac{v}{v^2+u^2} \end{matrix}\right], = \text{H11\_22:,v}]$$

(%o60) done

```
(%i61) seconda_specie(7,7,si);
```

[Simboli di Christoffel seconda specie,cartesian3d]

$$\left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right], = \text{H10\_22:,x}]$$

$$\left[\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right], = \text{H11\_22:,y}]$$

$$\left[\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\right], = \text{H12\_22:,z}]$$

(%o61) done

```
(%i62) seconda_specie(8,8,si);
```

[Simboli di Christoffel seconda specie,polarcylindrical]

$$\left[\begin{matrix}1 & 0 & 0\\ 0 & r^2 & 0\\ 0 & 0 & 1\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}0 & 0 & 0\\ 0 & -r & 0\\ 0 & 0 & 0\end{matrix}\right], = \text{H10\_22:,r}]$$

$$\left[\begin{matrix}0 & \frac{1}{r} & 0\\ \frac{1}{r} & 0 & 0\\ 0 & 0 & 0\end{matrix}\right], = \text{H11\_22:,}\theta]$$

$$\left[\begin{matrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{matrix}\right], = \text{H12\_22:,z}]$$

(%o62) done

(%i63) seconda_specie(9,9,si);

[Simboli di Christoffel seconda specie,paraboloidal]

$$\left[\begin{matrix} u^2\,v^2 & 0 & 0 \\ 0 & v^2+u^2 & 0 \\ 0 & 0 & v^2+u^2 \end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix} \frac{1}{u} & \frac{1}{v} & 0 \\ \frac{1}{v} & -\frac{1}{u\,v^2} & 0 \\ 0 & 0 & -\frac{1}{u\,v^2} \end{matrix}\right], = H10\_22:,u]$$

$$\left[\begin{matrix} -\frac{u^2\,v}{v^2+u^2} & \frac{u}{v^2+u^2} & 0 \\ \frac{u}{v^2+u^2} & \frac{v}{v^2+u^2} & 0 \\ 0 & 0 & -\frac{v}{v^2+u^2} \end{matrix}\right], = H11\_22:,v]$$

$$\left[\begin{matrix} 0 & 0 & \frac{u}{v^2+u^2} \\ 0 & 0 & \frac{v}{v^2+u^2} \\ \frac{u}{v^2+u^2} & \frac{v}{v^2+u^2} & 0 \end{matrix}\right], = H12\_22:,\phi]$$

(%o63) done

(%i64) seconda_specie(10,10,no1);

[Simboli di Christoffel seconda specie,conical]

Componenti del tensore metrico

$$\left[\frac{(v-u)(v+u)w^2}{(u-e)(u+e)(u-f)(u+f)},=,u,u\right]$$

$$\left[\frac{(u-v)(v+u)w^2}{(v-e)(v+e)(v-f)(v+f)},=,v,v\right]$$

$$[1,=,w,w]$$

Simboli di Christoffel di seconda specie

[Per la componente ,u]

$$\left[-\frac{(2u^3+(-f^2-e^2)u)v^2-u^5+e^2f^2u}{(u^4+(-f^2-e^2)u^2+e^2f^2)v^2-u^6+(f^2+e^2)u^4-e^2f^2u^2},=,u,u,u\right]$$

$$\left[\frac{v}{v^2-u^2},=,u,u,v\right]$$

$$\left[\frac{1}{w},=,u,u,w\right]$$

$$\left[-\frac{u^5+(-f^2-e^2)u^3+e^2f^2u}{v^6+(-u^2-f^2-e^2)v^4+((f^2+e^2)u^2+e^2f^2)v^2-e^2f^2u^2},=,u,v,v\right]$$

(%o64) done

(%i65) seconda_specie(10,10,no2);

[Simboli di Christoffel seconda specie,conical]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente ,v]

$$\left[\frac{v^5+(-f^2-e^2)v^3+e^2f^2v}{(u^4+(-f^2-e^2)u^2+e^2f^2)v^2-u^6+(f^2+e^2)u^4-e^2f^2u^2},=,v,u,u\right]$$

$$\left[-\frac{u}{v^2-u^2},=,v,u,v\right]$$

$$\left[-\frac{v^5-2u^2v^3+((f^2+e^2)u^2-e^2f^2)v}{v^6+(-u^2-f^2-e^2)v^4+((f^2+e^2)u^2+e^2f^2)v^2-e^2f^2u^2},=,v,v,v\right]$$

$$\left[\frac{1}{w},=,v,v,w\right]$$

(%o65) done

```
(%i66) seconda_specie(10,10,no3);
```

[Simboli di Christoffel seconda specie,conical]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,w]

$$\left[-\frac{(v^2-u^2)\,w}{u^4+(-f^2-e^2)\,u^2+e^2\,f^2},=,w,u,u\right]$$

$$\left[\frac{(v^2-u^2)\,w}{v^4+(-f^2-e^2)\,v^2+e^2\,f^2},=,w,v,v\right]$$

(%o66) *done*

```
(%i67) seconda_specie(11,11,no1);
```

[Simboli di Christoffel seconda specie,toroidal]
Componenti del tensore metrico

$$\left[\frac{e^2\,\sinh(v)^2}{(\cosh(v)-\cos(u))^2},=,u,u\right]$$

$$\left[\frac{e^2}{(\cosh(v)-\cos(u))^2},=,v,v\right]$$

$$\left[\frac{e^2}{(\cosh(v)-\cos(u))^2},=,\phi,\phi\right]$$

Simboli di Christoffel di seconda specie
[Per la componente ,u]

$$\left[-\frac{\sin(u)}{\cosh(v)-\cos(u)},=,u,u,u\right]$$

$$\left[-\frac{\cos(u)\cosh(v)-1}{(\cosh(v)-\cos(u))\sinh(v)},=,u,u,v\right]$$

$$\left[\frac{\sin(u)}{(\cosh(v)-\cos(u))\sinh(v)^2},=,u,v,v\right]$$

$$\left[\frac{\sin(u)}{(\cosh(v)-\cos(u))\sinh(v)^2},=,u,\phi,\phi\right]$$

(%o67) *done*

```
(%i68) seconda_specie(11,11,no2);
[Simboli di Christoffel seconda specie,toroidal]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,v]
                                (cos(u)cosh(v)-1)sinh(v)
[                               ------------------------,=,v,u,u]
                                     cosh(v)-cos(u)
                                      sin(u)
[                               - ------------,=,v,u,v]
                                  cosh(v)-cos(u)
                                     sinh(v)
[                               - ------------,=,v,v,v]
                                  cosh(v)-cos(u)
                                    sinh(v)
[                               ------------,=,v,ϕ,ϕ]
                                 cosh(v)-cos(u)
(%o68) done
```

```
(%i69) seconda_specie(11,11,no3);
[Simboli di Christoffel seconda specie,toroidal]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,ϕ]
                                  sin(u)
[                               - ------------,=,ϕ,u,ϕ]
                                  cosh(v)-cos(u)
                                     sinh(v)
[                               - ------------,=,ϕ,v,ϕ]
                                  cosh(v)-cos(u)
(%o69) done
```

```
(%i70) seconda_specie(12,12,si);
```

[Simboli di Christoffel seconda specie,spherical]

$$\left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 \end{bmatrix}, = \text{Tensore metrico g22}\right]$$

$$\left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & -r & 0 \\ 0 & 0 & -r\sin(\theta)^2 \end{bmatrix}, = H10\_22:, r\right]$$

$$\left[\begin{bmatrix} 0 & \frac{1}{r} & 0 \\ \frac{1}{r} & 0 & 0 \\ 0 & 0 & -\cos(\theta)\sin(\theta) \end{bmatrix}, = H11\_22:, \theta\right]$$

$$\left[\begin{bmatrix} 0 & 0 & \frac{1}{r} \\ 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{bmatrix}, = H12\_22:, \phi\right]$$

(%o70) done

```
(%i71) seconda_specie(13,13,no1);
```

[Simboli di Christoffel seconda specie,oblatespheroidal]

Componenti del tensore metrico

$$[e^2(\sin(v)^2+\sinh(u)^2), =, u, u]$$

$$[e^2(\sin(v)^2+\sinh(u)^2), =, v, v]$$

$$[e^2\cosh(u)^2\cos(v)^2, =, \phi, \phi]$$

Simboli di Christoffel di seconda specie

[Per la componente ,u]

$$\left[\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2}, =, u, u, u\right]$$

$$\left[\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2}, =, u, u, v\right]$$

$$\left[-\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2}, =, u, v, v\right]$$

$$\left[-\frac{\cosh(u)\sinh(u)\cos(v)^2}{\sin(v)^2+\sinh(u)^2}, =, u, \phi, \phi\right]$$

(%o71) done

```
(%i72) seconda_specie(13,13,no2);
```

[Simboli di Christoffel seconda specie,oblatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,v]

$$\left[-\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,v,u,u\right]$$

$$\left[\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2},=,v,u,v\right]$$

$$\left[\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,v,v,v\right]$$

$$\left[\frac{\cosh(u)^2\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,v,\phi,\phi\right]$$

(%o72) *done*

```
(%i73) seconda_specie(13,13,no3);
```

[Simboli di Christoffel seconda specie,oblatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,ϕ]

$$\left[\frac{\sinh(u)}{\cosh(u)},=,\phi,u,\phi\right]$$

$$\left[-\frac{\sin(v)}{\cos(v)},=,\phi,v,\phi\right]$$

(%o73) *done*

(%i74) seconda_specie(14,14,no1);

[Simboli di Christoffel seconda specie,oblatespheroidalsqrt]

Componenti del tensore metrico

$$\left[\frac{e^2\left(u^2-v^2\right)}{u^2-1},=,u,u\right]$$

$$\left[\frac{e^2\left(u^2-v^2\right)}{u^2-1},=,v,v\right]$$

$$\left[e^2\,u^2\,v^2,=,\phi,\phi\right]$$

Simboli di Christoffel di seconda specie

[Per la componente ,u]

$$\left[-\frac{u\,v^2-u}{\left(u^2-1\right)v^2-u^4+u^2},=,u,u,u\right]$$

$$\left[\frac{v}{v^2-u^2},=,u,u,v\right]$$

$$\left[\frac{u\,v^2-u}{\left(u^2-1\right)v^2-u^4+u^2},=,u,v,v\right]$$

$$\left[\frac{\left(u^3-u\right)v^2}{v^2-u^2},=,u,\phi,\phi\right]$$

(%o74) done

(%i75) seconda_specie(14,14,no2);

[Simboli di Christoffel seconda specie,oblatespheroidalsqrt]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente ,v]

$$\left[-\frac{v}{v^2-u^2},=,v,u,u\right]$$

$$\left[-\frac{u\,v^2-u}{\left(u^2-1\right)v^2-u^4+u^2},=,v,u,v\right]$$

$$\left[\frac{v}{v^2-u^2},=,v,v,v\right]$$

$$\left[\frac{\left(u^4-u^2\right)v}{v^2-u^2},=,v,\phi,\phi\right]$$

(%o75) done

```
(%i76) seconda_specie(14,14,no3);
```

[Simboli di Christoffel seconda specie,oblatespheroidalsqrt]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente ,ϕ]

$$\left[\frac{1}{u},=,\phi,u,\phi\right]$$

$$\left[\frac{1}{v},=,\phi,v,\phi\right]$$

```
(%o76) done
```

```
(%i77) seconda_specie(15,15,no1);
```

[Simboli di Christoffel seconda specie,prolatespheroidal]

Componenti del tensore metrico

$$\left[e^2\left(\sin(v)^2+\sinh(u)^2\right),=,u,u\right]$$

$$\left[e^2\left(\sin(v)^2+\sinh(u)^2\right),=,v,v\right]$$

$$\left[e^2\sinh(u)^2\sin(v)^2,=,\phi,\phi\right]$$

Simboli di Christoffel di seconda specie

[Per la componente ,u]

$$\left[\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2},=,u,u,u\right]$$

$$\left[\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,u,u,v\right]$$

$$\left[-\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2},=,u,v,v\right]$$

$$\left[-\frac{\cosh(u)\sinh(u)\sin(v)^2}{\sin(v)^2+\sinh(u)^2},=,u,\phi,\phi\right]$$

```
(%o77) done
```

```
(%i78) seconda_specie(15,15,no2);
```

[Simboli di Christoffel seconda specie,prolatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,v]

$$\left[-\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,v,u,u\right]$$

$$\left[\frac{\cosh(u)\sinh(u)}{\sin(v)^2+\sinh(u)^2},=,v,u,v\right]$$

$$\left[\frac{\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,v,v,v\right]$$

$$\left[-\frac{\sinh(u)^2\cos(v)\sin(v)}{\sin(v)^2+\sinh(u)^2},=,v,\phi,\phi\right]$$

(%o78) *done*

```
(%i79) seconda_specie(15,15,no3);
```

[Simboli di Christoffel seconda specie,prolatespheroidal]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente ,ϕ]

$$\left[\frac{\cosh(u)}{\sinh(u)},=,\phi,u,\phi\right]$$

$$\left[\frac{\cos(v)}{\sin(v)},=,\phi,v,\phi\right]$$

(%o79) *done*

```
(%i80) seconda_specie(16,16,si);
[Simboli di Christoffel seconda specie,prolatespheroidalsqrt]
```

$$\left[\begin{matrix}\frac{e^2\,(v^2-u^2)}{1-u^2} & 0 & 0\\ 0 & \frac{e^2\,(v^2-u^2)}{v^2-1} & 0\\ 0 & 0 & e^2\,(1-u^2)\,(v^2-1)\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}-\frac{u\,v^2-u}{(u^2-1)\,v^2-u^4+u^2} & \frac{v}{v^2-u^2} & 0\\ \frac{v}{v^2-u^2} & -\frac{u^3-u}{v^4+(-u^2-1)\,v^2+u^2} & 0\\ 0 & 0 & -\frac{(u^3-u)\,v^2-u^3+u}{v^2-u^2}\end{matrix}\right], = H10\_22:,u]$$

$$\left[\begin{matrix}\frac{v^3-v}{(u^2-1)\,v^2-u^4+u^2} & -\frac{u}{v^2-u^2} & 0\\ -\frac{u}{v^2-u^2} & \frac{(u^2-1)\,v}{v^4+(-u^2-1)\,v^2+u^2} & 0\\ 0 & 0 & \frac{(u^2-1)\,v^3+(1-u^2)\,v}{v^2-u^2}\end{matrix}\right], = H11\_22:,v]$$

$$\left[\begin{matrix}0 & 0 & \frac{u}{u^2-1}\\ 0 & 0 & \frac{v}{v^2-1}\\ \frac{u}{u^2-1} & \frac{v}{v^2-1} & 0\end{matrix}\right], = H12\_22:,\phi]$$

```
(%o80) done
```

```
(%i81) seconda_specie(17,17,no1);
```

[Simboli di Christoffel seconda specie,ellipsoidal]

Componenti del tensore metrico

$$[(b^2\sin(\phi)^2+a^2\cos(\phi)^2)\sin(\theta)^2+c^2\cos(\theta)^2,=,r,r]$$

$$[(b^2\sin(\phi)^2+a^2\cos(\phi)^2-c^2)\,r\cos(\theta)\sin(\theta),=,r,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)\,r\sin(\theta)^2,=,r,\phi]$$

$$[r^2\left(c^2\sin(\theta)^2+(b^2\sin(\phi)^2+a^2\cos(\phi)^2)\cos(\theta)^2\right),=,\theta,\theta]$$

$$[(b^2-a^2)\cos(\phi)\sin(\phi)\,r^2\cos(\theta)\sin(\theta),=,\theta,\phi]$$

$$[(a^2\sin(\phi)^2+b^2\cos(\phi)^2)\,r^2\sin(\theta)^2,=,\phi,\phi]$$

Simboli di Christoffel di seconda specie

[Per la componente ,r]

$$[-r,=,r,\theta,\theta]$$

$$[-r\sin(\theta)^2,=,r,\phi,\phi]$$

(%o81) *done*

```
(%i82) seconda_specie(17,17,no2);
```

[Simboli di Christoffel seconda specie,ellipsoidal]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente ,θ]

$$[\frac{1}{r},=,\theta,r,\theta]$$

$$[-\cos(\theta)\sin(\theta),=,\theta,\phi,\phi]$$

(%o82) *done*

```
(%i83) seconda_specie(17,17,no3);
```

[Simboli di Christoffel seconda specie,ellipsoidal]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente ,ϕ]

$$[\frac{1}{r},=,\phi,r,\phi]$$

$$[\frac{\cos(\theta)}{\sin(\theta)},=,\phi,\theta,\phi]$$

(%o83) *done*

```
(%i84) seconda_specie(18,18,si);
[Simboli di Christoffel seconda specie,cartesian4d]
```

$$\left[\begin{matrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{matrix}\right], = \textit{Tensore metrico g22}]$$

$$\left[\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{matrix}\right], = H10\_22:,x]$$

$$\left[\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{matrix}\right], = H11\_22:,y]$$

$$\left[\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{matrix}\right], = H12\_22:,z]$$

$$\left[\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{matrix}\right], = H13\_22:,t]$$

```
(%o84) done
```

(%i85) seconda_specie(19,19,si);

[Simboli di Christoffel seconda specie,spherical4d]

$$\left[\begin{matrix}1 & 0 & 0 & 0\\ 0 & r^2 & 0 & 0\\ 0 & 0 & r^2\sin(\theta)^2 & 0\\ 0 & 0 & 0 & \sin(\eta)^2 r^2\sin(\theta)^2\end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix}0 & 0 & 0 & 0\\ 0 & -r & 0 & 0\\ 0 & 0 & -r\sin(\theta)^2 & 0\\ 0 & 0 & 0 & -\sin(\eta)^2 r\sin(\theta)^2\end{matrix}\right], = \text{H10\_22:,r}]$$

$$\left[\begin{matrix}0 & \frac{1}{r} & 0 & 0\\ \frac{1}{r} & 0 & 0 & 0\\ 0 & 0 & -\cos(\theta)\sin(\theta) & 0\\ 0 & 0 & 0 & -\sin(\eta)^2\cos(\theta)\sin(\theta)\end{matrix}\right], = \text{H11\_22:,}\theta]$$

$$\left[\begin{matrix}0 & 0 & \frac{1}{r} & 0\\ 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} & 0\\ \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 & 0\\ 0 & 0 & 0 & -\cos(\eta)\sin(\eta)\end{matrix}\right], = \text{H12\_22:,}\eta]$$

$$\left[\begin{matrix}0 & 0 & 0 & \frac{1}{r}\\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)}\\ 0 & 0 & 0 & \frac{\cos(\eta)}{\sin(\eta)}\\ \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & \frac{\cos(\eta)}{\sin(\eta)} & 0\end{matrix}\right], = \text{H13\_22:,}\phi]$$

(%o85) done

```
(%i86) seconda_specie(20,20,si);
```

[Simboli di Christoffel seconda specie,exteriorschwarzschild]

$$\left[\begin{pmatrix} \frac{2\,m-r}{r} & 0 & 0 & 0 \\ 0 & \frac{r}{r-2\,m} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)^2 \end{pmatrix}, = \text{Tensore metrico g22}\right]$$

$$\left[\begin{pmatrix} 0 & \frac{m}{r^2-2\,m\,r} & 0 & 0 \\ \frac{m}{r^2-2\,m\,r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, = H10\_22:,t\right]$$

$$\left[\begin{pmatrix} \frac{m\,r-2\,m^2}{r^3} & 0 & 0 & 0 \\ 0 & -\frac{m}{r^2-2\,m\,r} & 0 & 0 \\ 0 & 0 & 2\,m-r & 0 \\ 0 & 0 & 0 & (2\,m-r)\sin(\theta)^2 \end{pmatrix}, = H11\_22:,r\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\sin(\theta) \end{pmatrix}, = H12\_22:,\theta\right]$$

$$\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{pmatrix}, = H13\_22:,\phi\right]$$

(%o86) *done*

(%i87) seconda_specie(21,21,si);

[Simboli di Christoffel seconda specie,interiorschwarzschild]

$$\left[\begin{matrix} -\frac{t}{2\,m-t} & 0 & 0 & 0 \\ 0 & \frac{2\,m-t}{t} & 0 & 0 \\ 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & t^2\sin(u)^2 \end{matrix}\right], = \text{Tensore metrico g22}]$$

$$\left[\begin{matrix} -\frac{m}{t^2-2\,m\,t} & 0 & 0 & 0 \\ 0 & \frac{m\,t-2\,m^2}{t^3} & 0 & 0 \\ 0 & 0 & 2\,m-t & 0 \\ 0 & 0 & 0 & (2\,m-t)\sin(u)^2 \end{matrix}\right], = H10\_22:,t]$$

$$\left[\begin{matrix} 0 & \frac{m}{t^2-2\,m\,t} & 0 & 0 \\ \frac{m}{t^2-2\,m\,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}\right], = H11\_22:,z]$$

$$\left[\begin{matrix} 0 & 0 & \frac{1}{t} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cos(u)\sin(u) \end{matrix}\right], = H12\_22:,u]$$

$$\left[\begin{matrix} 0 & 0 & 0 & \frac{1}{t} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos(u)}{\sin(u)} \\ \frac{1}{t} & 0 & \frac{\cos(u)}{\sin(u)} & 0 \end{matrix}\right], = H13\_22:,v]$$

(%o87) done

```
(%i88) seconda_specie(22,22,no1);
```

[Simboli di Christoffel seconda specie, kerr_newman]

Componenti del tensore metrico

$$\left[\frac{a^2\sin(\theta)^2-r^2+2\,m\,r-e^2-a^2}{a^2\cos(\theta)^2+r^2},=,ct,ct\right]$$

$$\left[\frac{a\,(e^2-2\,m\,r)\sin(\theta)^2}{a^2\cos(\theta)^2+r^2},=,ct,\phi\right]$$

$$\left[\frac{a^2\cos(\theta)^2+r^2}{r^2-2\,m\,r+e^2+a^2},=,r,r\right]$$

$$\left[a^2\cos(\theta)^2+r^2,=,\theta,\theta\right]$$

$$\left[\frac{\sin(\theta)^2\,((r^2+a^2)^2-a^2\,(r^2-2\,m\,r+e^2+a^2)\sin(\theta)^2)}{a^2\cos(\theta)^2+r^2},=,\phi,\phi\right]$$

Simboli di Christoffel di seconda specie

[Per la componente ,ct]

$$[-((a^2\,m\,r^2+a^4\,m)\cos(\theta)^2-m\,r^4+e^2\,r^3-a^2\,m\,r^2+a^2\,e^2\,r)/($$
$$(a^4\,r^2-2\,a^4\,m\,r+a^4\,e^2+a^6)\cos(\theta)^4+(2\,a^2\,r^4-4\,a^2\,m\,r^3+(2\,a^2\,e^2+2\,a^4)\,r^2)\cos(\theta)^2+r^6$$
$$-2\,m\,r^5+(e^2+a^2)\,r^4),=,ct,ct,r]$$

$$\left[-\frac{(2\,a^2\,m\,r-a^2\,e^2)\cos(\theta)\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,ct,ct,\theta\right]$$

$$[((a^3\,m\,r^2-a^3\,e^2\,r-a^5\,m)\cos(\theta)^4+(3\,a\,m\,r^4-2\,a\,e^2\,r^3+a^5\,m)\cos(\theta)^2-3\,a\,m\,r^4+2\,a\,e^2$$
$$r^3-a^3\,m\,r^2+a^3\,e^2\,r)/((a^4\,r^2-2\,a^4\,m\,r+a^4\,e^2+a^6)\cos(\theta)^4+$$
$$(2\,a^2\,r^4-4\,a^2\,m\,r^3+(2\,a^2\,e^2+2\,a^4)\,r^2)\cos(\theta)^2+r^6-2\,m\,r^5+(e^2+a^2)\,r^4),=,ct,r,\phi]$$

$$\left[-\frac{((2\,a^3\,m\,r-a^3\,e^2)\cos(\theta)^3+(a^3\,e^2-2\,a^3\,m\,r)\cos(\theta))\sin(\theta)}{a^4\cos(\theta)^4+2\,a^2\,r^2\cos(\theta)^2+r^4},=,ct,\theta,\phi\right]$$

```
(%o88) done
```

```
(%i89) seconda_specie(22,22,no2);
```

[Simboli di Christoffel seconda specie, kerr_newman]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente , r]

$$[((a^2 r^3-2 a^2 m r^2+(a^2 e^2+a^4) r)\sin(\theta)^2+(a^2 r^3-3 a^2 m r^2+(2 a^2 m^2+a^2 e^2+a^4) r+(-a^2 e^2-a^4) m)\cos(\theta)^2+m r^4+(-2 m^2-e^2-a^2) r^3+(3 e^2+3 a^2) m r^2+(-e^4-2 a^2 e^2-a^4) r)/(a^6 \cos(\theta)^6+3 a^4 r^2 \cos(\theta)^4+3 a^2 r^4 \cos(\theta)^2+r^6), =, r, ct, ct]$$

$$[(((a^3 m r^4-4 a^3 m^2 r^3+(4 a^3 m^3+(2 a^3 e^2+2 a^5) m) r^2+(-4 a^3 e^2-4 a^5) m^2 r+(a^3 e^4+2 a^5 e^2+a^7) m)\cos(\theta)^2-a m r^6+(4 a m^2+a e^2) r^5+((-6 a e^2-2 a^3) m-4 a m^3) r^4+((8 a e^2+4 a^3) m^2+2 a e^4+2 a^3 e^2) r^3+(-5 a e^4-6 a^3 e^2-a^5) m r^2+(a e^6+2 a^3 e^4+a^5 e^2) r)\sin(\theta)^2)/((a^6 r^2-2 a^6 m r+a^6 e^2+a^8)\cos(\theta)^6+(3 a^4 r^4-6 a^4 m r^3+(3 a^4 e^2+3 a^6) r^2)\cos(\theta)^4+(3 a^2 r^6-6 a^2 m r^5+(3 a^2 e^2+3 a^4) r^4)\cos(\theta)^2+r^8-2 m r^7+(e^2+a^2) r^6), =, r, ct, \phi]$$

$$[-\frac{(a^2 r-a^2 m)\cos(\theta)^2+m r^2+(-e^2-a^2) r}{(a^2 r^2-2 a^2 m r+a^2 e^2+a^4)\cos(\theta)^2+r^4-2 m r^3+(e^2+a^2) r^2}, =, r, r, r]$$

$$[-\frac{a^2 \cos(\theta)\sin(\theta)}{a^2 \cos(\theta)^2+r^2}, =, r, r, \theta]$$

$$[-\frac{r^3-2 m r^2+(e^2+a^2) r}{a^2 \cos(\theta)^2+r^2}, =, r, \theta, \theta]$$

$$[((a^4 r^3-3 a^4 m r^2+(2 a^4 m^2+a^4 e^2+a^6) r+(-a^4 e^2-a^6) m)\cos(\theta)^2+a^2 m r^4+(-2 a^2 m^2-a^2 e^2-a^4) r^3+(3 a^2 e^2+3 a^4) m r^2+(-a^2 e^4-2 a^4 e^2-a^6) r)\sin(\theta)^4+((-2 a^2 r^5+4 a^2 m r^4+(-2 a^2 e^2-4 a^4) r^3+4 a^4 m r^2+(-2 a^4 e^2-2 a^6) r)\cos(\theta)^2-r^7+2 m r^6+(-e^2-a^2) r^5+a^4 r^3-2 a^4 m r^2+(a^4 e^2+a^6) r)\sin(\theta)^2)/(a^6 \cos(\theta)^6+3 a^4 r^2 \cos(\theta)^4+3 a^2 r^4 \cos(\theta)^2+r^6), =, r, \phi, \phi]$$

```
(%o89) done
```

```
(%i90) seconda_specie(22,22,no3);
```

[Simboli di Christoffel seconda specie,kerr_newman]

Componenti del tensore metrico

Simboli di Christoffel di seconda specie

[Per la componente ,θ]

$$\left[-\frac{\left(2\,a^2\,m\,r-a^2\,e^2\right)\cos(\theta)\sin(\theta)}{a^6\cos(\theta)^6+3\,a^4\,r^2\cos(\theta)^4+3\,a^2\,r^4\cos(\theta)^2+r^6},=,\theta,ct,ct\right]$$

$$\left[\frac{\left(2\,a\,m\,r^3-a\,e^2\,r^2+2\,a^3\,m\,r-a^3\,e^2\right)\cos(\theta)\sin(\theta)}{a^6\cos(\theta)^6+3\,a^4\,r^2\cos(\theta)^4+3\,a^2\,r^4\cos(\theta)^2+r^6},=,\theta,ct,\phi\right]$$

$$\left[\frac{a^2\cos(\theta)\sin(\theta)}{\left(a^2\,r^2-2\,a^2\,m\,r+a^2\,e^2+a^4\right)\cos(\theta)^2+r^4-2\,m\,r^3+\left(e^2+a^2\right)r^2},=,\theta,r,r\right]$$

$$\left[\frac{r}{a^2\cos(\theta)^2+r^2},=,\theta,r,\theta\right]$$

$$\left[-\frac{a^2\cos(\theta)\sin(\theta)}{a^2\cos(\theta)^2+r^2},=,\theta,\theta,\theta\right]$$

$$\left[-\left(\left(\left(a^4\,r^2-2\,a^4\,m\,r+a^4\,e^2+a^6\right)\cos(\theta)^5+\left(2\,a^2\,r^4-4\,a^2\,m\,r^3+\left(2\,a^2\,e^2+2\,a^4\right)r^2\right)\cos(\theta)^3+\left(r^6+a^2\,r^4+4\,a^2\,m\,r^3-2\,a^2\,e^2\,r^2+2\,a^4\,m\,r-a^4\,e^2\right)\cos(\theta)\right)\sin(\theta)\right)/\left(a^6\cos(\theta)^6+3\,a^4\,r^2\cos(\theta)^4+3\,a^2\,r^4\cos(\theta)^2+r^6\right),=,\theta,\phi,\phi\right]$$

```
(%o90) done
```

```
(%i91) seconda_specie(22,22,no4);
```

[Simboli di Christoffel seconda specie, kerr_newman]
Componenti del tensore metrico
Simboli di Christoffel di seconda specie
[Per la componente , ϕ]

$$[-(a^3 m \cos(\theta)^2 - a m r^2 + a e^2 r)/((a^4 r^2 - 2 a^4 m r + a^4 e^2 + a^6)\cos(\theta)^4 + (2 a^2 r^4 - 4 a^2 m r^3 + (2 a^2 e^2 + 2 a^4) r^2)\cos(\theta)^2 + r^6 - 2 m r^5 + (e^2 + a^2) r^4), =, \phi, ct, r]$$

$$[-\frac{(2 a m r - a e^2)\cos(\theta)}{(a^4 \cos(\theta)^4 + 2 a^2 r^2 \cos(\theta)^2 + r^4)\sin(\theta)}, =, \phi, ct, \theta]$$

$$[((a^4 r - a^4 m)\cos(\theta)^4 + (2 a^2 r^3 - a^2 m r^2 + a^4 m)\cos(\theta)^2 + r^5 - 2 m r^4 + e^2 r^3 - a^2 m r^2 + a^2 e^2 r)/((a^4 r^2 - 2 a^4 m r + a^4 e^2 + a^6)\cos(\theta)^4 + (2 a^2 r^4 - 4 a^2 m r^3 + (2 a^2 e^2 + 2 a^4) r^2)\cos(\theta)^2 + r^6 - 2 m r^5 + (e^2 + a^2) r^4), =, \phi, r, \phi]$$

$$[((a^4 r^2 - 2 a^4 m r + a^4 e^2 + a^6)\cos(\theta)^5 + (2 a^2 r^4 - 6 a^2 m r^3 + (4 a^2 m^2 + 3 a^2 e^2 + 2 a^4) r^2 + (-4 a^2 e^2 - 2 a^4) m r + a^2 e^4 + a^4 e^2)\cos(\theta)^3 + (r^6 - 2 m r^5 + (e^2 + a^2) r^4 + 2 a^2 m r^3 + (-4 a^2 m^2 - a^2 e^2) r^2 + (4 a^2 e^2 + 2 a^4) m r - a^2 e^4 - a^4 e^2)\cos(\theta))/((a^4 r^2 - 2 a^4 m r + a^4 e^2 + a^6)\cos(\theta)^4 + (2 a^2 r^4 - 4 a^2 m r^3 + (2 a^2 e^2 + 2 a^4) r^2)\cos(\theta)^2 + r^6 - 2 m r^5 + (e^2 + a^2) r^4)\sin(\theta)), =, \phi, \theta, \phi]$$

```
(%o91) done
```

```
Le formule per la metrica di kerr-newman sono complicatissime.
Se pero' il buco nero non ruota diventano molto più semplici...
Si tratta in tal caso della soluzione di Reissner e Nordstrom.
```

```
(%i92) ev(lg,a=0);
```

$$(\%o92)\quad \begin{bmatrix} \frac{-r^2 + 2 m r - e^2}{r^2} & 0 & 0 & 0 \\ 0 & \frac{r^2}{r^2 - 2 m r + e^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}$$

(%i93) `ev(ratsimp(mcs_m[1]),a=0);`

(%o93)
$$\begin{bmatrix} 0 & \frac{m\,r-e^2}{r^3-2\,m\,r^2+e^2\,r} & 0 & 0 \\ \frac{m\,r-e^2}{r^3-2\,m\,r^2+e^2\,r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(%i94) `ev(ratsimp(mcs_m[2]),a=0);`

(%o94)
$$\begin{bmatrix} \frac{m\,r^3+(-2\,m^2-e^2)\,r^2+3\,e^2\,m\,r-e^4}{r^5} & 0 & 0 & 0 \\ 0 & -\frac{m\,r-e^2}{r^3-2\,m\,r^2+e^2\,r} & 0 & 0 \\ 0 & 0 & -\frac{r^2-2\,m\,r+e^2}{r} & 0 \\ 0 & 0 & 0 & -\frac{(r^2-2\,m\,r+e^2)\,\sin(\theta)^2}{r} \end{bmatrix}$$

(%i95) `ev(ratsimp(mcs_m[3]),a=0);`

(%o95)
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\sin(\theta) \end{bmatrix}$$

(%i96) `ev(ratsimp(mcs_m[4]),a=0);`

(%o96)
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{bmatrix}$$